

AXIOMATIC IMPERATIVES: A PROOF-THEORETIC LOGIC OF SATISFACTION

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Issues in Dynamic Semantics
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OUTLINE

- 1 BACKGROUND
 - Introduction
 - Some Issues
 - Existing accounts
- 2 THE PROPOSAL
 - Overview of Theory
 - The Formal Theory
 - Some Issues
- 3 CONCLUSIONS & FUTURE WORK
 - Overview of paradigm
 - Summary
 - Future Work

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IMPERATIVES

The Issue

How can be best capture the 'logical' behaviour of imperatives:

- the patterns of “entailment” between imperatives;
 - the relationship between imperatives and propositions?
-
- To satisfy the demand to “*swim or run!*” you must either swim or run, and so satisfy the demand “*swim!*” or “*run!*”

FORMALISING IMPERATIVES

An old question, but there is

- no agreed, fully worked-out basic theory, even for paradigm cases (contrast with propositions);
- some disagreement even what the appropriate behaviours are, even in apparently elementary cases.

E.g. is the previous example

— going from “*swim or run!*” to “*swim!*” (or “*run!*”) —
even an inference, as such?

IMPERATIVES & DYNAMIC SEMANTICS

- In order to analyse imperatives, we are required to consider a dynamic world that changes and can be changed.
- We could seek to formalise the behaviour of imperatives in terms of actions, events, causes and intentions.
 - Formalising and modelling actions, events and causation is not straightforward.
- How far we can go in capturing a core set of patterns of behaviour for paradigm cases of imperatives at a level of abstraction that does not include actions & events as such?

AIMS AND SCOPE

- To develop a proof theoretic semantics (and associated model-theory) that captures paradigm “entailment relationships” involving imperatives.
- Avoid a reductive analysis
 - of imperative to propositions
 - *imperatives are of a different category,*
 - of actions / events / intent / causation
 - *these notions can be problematic.*
- Avoid ad hoc machinery for generic problems
 - *such as time, causation, and other general issues.*
- *Not addressing (i) overtly “pragmatic” issues, (ii) details of the syntax-semantics interface.*

THE DATA

- Initially focus on paradigm cases of imperatives.
 - Utterances that are imperative in both form and use.
 - *“Shut the door!”*
- “logical” compositions of such imperatives.

CONJUNCTION *“Close the window and open the door!”*

DISJUNCTION A: [*“What do I have to do?”*]

B: *“Prepare the lecture or mark the exams. . .*

(i) . . . *it’s up to you.*” (Free Choice)

(ii) . . . *we don’t yet know which.*” (Weak)

NEGATION *“Don’t come to the party!”*

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SOME COMMON PROBLEMS

- Negation (meaning, and kinds)
- Disjunction (inferences, and kinds)
 - Ross' Paradox & Disjunction Introduction v Disjunction Elimination
- *[Other phenomena, e.g. Conditional Imperatives and Pseudo Imperatives]*

▶▶ skip details for the moment

NEGATIVE ACTIONS

- *“Don’t buy an apple!”*
Appears to require us to quantify over all actions of an agent and their consequences

TYPES OF NEGATION

- “Don’t come to the party!” Does this mean
 “Don’t be present at the party!”

or

“Don’t take steps to be at the party!”?

[Hamblin 87]

ROSS' PARADOX

- From

“Post the letter”

a logic of satisfaction that mirrors conventional propositional logic will allow us to infer

“Post the letter or burn the letter!”

(i.e. $a \vdash (a \vee b)$) and not the other way around.

- This seems at odds with intuitions, which suggest the inference should just go the other way ($(a \vee b) \vdash a$) [Kamp 73].

WEAK DISJUNCTION

- In weak disjunction, there is an “order” to do ($a \vee b$), but there is no free-choice, it is just not yet clear which it should be.
- How do we formally distinguish between free-choice and weak disjunction, and what are the appropriate semantics?

SOME OTHER ISSUES

All of the following are problematic.

- Combinations of imperatives and propositions
 - conditional imperatives,
 - pseudo-imperatives,
 - “practical reasoning”.
- Issues of temporal ordering of conjunctive imperatives
- Permission revision
- Pragmatics in general
 - What do you do you intend to happen following an utterance
 - What do you do when you hear an utterance

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SUMMARY OF EXISTING ACCOUNTS

Existing accounts typically use some combination of the following approaches.



- 1 Reduce imperatives to propositions**
(to explain “inferences” between imperative).
- 2 Assume a theory of actions and events**
(to express what has been requested and describe when an imperative is satisfied).
- 3 Adopt a variant of a possible worlds model**
(to capture the notion of changing worlds updated by actions).
- 4 Appeal to conversational maxims**
(to explain disjunctive inferences, and the different disjunctive readings).

SOME PROBLEMS

In outline.

- 1 **Reductive analysis:** Imperatives are not propositions (*they do not have truth conditions*).
- 2 **Actions/Events:** These can be problematic (*e.g. causation, intensionality, frame problem, individuation*).
- 3 **Possible Worlds:** Some issues in how they are used (*e.g. notion of action is assumed, but outside the realm of individual possible worlds, and a notion of “closest world” sometimes required*).
- 4 **Maxims:** Would prefer conversational maxims to be the preserve of pragmatics, not semantics.

Many existing accounts are “incomplete” in their detail.

THE WAY FORWARD

Desiderata

Develop a semantic theory of imperatives that

- 1 is not overtly reductive,
- 2 does not assume a theory of actions/events,
- 3 or possible worlds,
- 4 and which does not rely upon conversational maxims.

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OVERVIEW

- Essentially, this is a formalisation of a “Logic of Satisfaction” [Hare 67].
- Capture the formal ‘logical’ behaviour of imperatives by way of inference rules over their propositional *satisfaction criteria*. (cf *fulfilment conditions, outcomes*)
- Not all the details to be presented here.
- Focusing on some key issues:
 - the notion of satisfying an imperative;
 - disjunction;
 - negation;
 - [*conditionals and pseudo-imperatives*]

SATISFACTION

- Satisfaction criteria: propositional *descriptions* of the relevant actions *by the relevant agent(s)*.
- “*Shut the door!*” (addressed to John) satisfied by the action description “*John shuts the door.*” in the salient future.
 - Action *per se* do not appear in the formalisation.
 - Actions are not specified by non-agentive extensional outcomes (“*The door is shut.*”)
- This side-steps problems with causation and related difficulties that can arise if we have actions, as such, in the semantic account.

ORGANISATION OF THEORY

We have the following

- some basic types;
- some “logical” operators;
- typing rules that assign types to complex expressions;
- rules for determining the truth conditions of complex propositions;
- rules for determining the satisfaction conditions of complex imperative.

In the following, we just exemplify some of the key aspects.

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TYPES

Imp_{Abs} — *Abstract Imperatives (un-uttered & undirected)*

Ag — *Agents (addressees)*

Imp — *Imperatives (uttered & directed)*

Prop — *Propositions*

- The usual logical connectives are assumed for propositions (Prop), with typing rules indicating which propositions are well formed, and proof rules for truth conditions.
- Variants of these connectives are assumed for imperative (Imp) and abstract imperatives (Imp_{Abs}) as required, with additional rules indicating which expressions are well-formed imperatives, together with proof rules that govern the satisfaction criteria.

SAMPLE TYPING RULES

BASIC RULES

$$\frac{j : \text{Imp}_{\text{Abs}} \quad a : \text{Ag}}{j![a] : \text{Imp}} \text{Imp}+ \quad \frac{j : \text{Imp}_{\text{Abs}} \quad a : \text{Ag}}{j![a]^{\downarrow} : \text{Prop}} \text{Prop}+$$

- \downarrow maps a directed imperative into a propositional description of its satisfaction criteria (in the present tense).
- The application of an abstract imperative (Imp_{Abs}) to an agent gives rise to imperative force.
This makes it more convenient to model Free Choice disjunction

FUTURE TENSE

- Imperatives are satisfied by actions that take place in some salient future.
 - Cf. the “fulfilment conditions” [Lappin 82] or “outcomes” [Ginzburg & Sag 2000]
- As such, the propositional descriptions of these actions are usually in the future tense

FUTURE TENSE Write Fp for p is true in the some (salient) future period.

$$\frac{p : \text{Prop}}{Fp : \text{Prop}}$$

SATISFACTION (ATOMIC)

SATISFACTION (ATOMIC)

$$\frac{j! : \text{Imp}_{\text{Abs}} \quad a : \text{Ag}}{Fj![a]^{\downarrow} \triangleright j![a]}$$

We shall write $p(a)$ for $Fp![a]^{\downarrow}$, so

Basic Satisfaction

$$\frac{p : \text{Imp}_{\text{Abs}} \quad a : \text{Ag}}{p(a) \triangleright p![a]}$$

“[In the future] John closes the door” \triangleright *“Close the door [John]!”*

SATISFACTION BY WAY OF LOGICAL CONSEQUENCE

$$\frac{p, q : \text{Prop} \quad i : \text{Imp} \quad q \vdash p \quad p \triangleright i}{q \triangleright i}$$

Eliding typing constraints from now on.

DISSATISFACTION

DISSATISFACTION

$$\frac{q \not\vdash p(a)}{q \not\vdash p![a]}$$

- If q does not allow us to derive the fact that “(In the future) John closes the door.”, then $q \not\vdash$ “Close the door [John]!”
- The rule is concerned with when q fails to allow us to infer that the imperative is satisfied (regardless of whether John complied with the imperative).

CONJUNCTION

TYPING

$$\frac{i, i' : \text{Imp}}{i \wedge i' : \text{Imp}}$$

SATISFACTION

$$\frac{p \triangleright i \quad p' \triangleright i'}{(p \wedge p') \triangleright (i \wedge i')} \wedge +$$

$$\frac{p \triangleright (i \wedge i')}{p \triangleright i} \wedge -_l \quad \frac{p \triangleright (i \wedge i')}{p \triangleright i'} \wedge -_r$$

Issue concerning whether $(p \wedge p')$ is realisable.

DISJUNCTION

TYPING

$$\frac{j, j' : \text{Imp}_{\text{Abs}}}{j \hat{\vee} j' : \text{Imp}_{\text{Abs}}} \quad \frac{i, i' : \text{Imp}}{i \dot{\vee} i' : \text{Imp}}$$

SATISFACTION (FREE CHOICE)

$$\frac{p \triangleright j![a]}{p \triangleright (j \hat{\vee} j')![a]} FC \hat{\vee} +_l \quad \frac{p' \triangleright j'[a]}{p' \triangleright (j \hat{\vee} j')![a]} FC \hat{\vee} +_r$$

$$\frac{p \triangleright (j \hat{\vee} j')![a]}{p \triangleright j![a] \vee p \triangleright j'![a]} FC \hat{\vee} -$$

WEAK DISJUNCTION

SATISFACTION (NON FREE CHOICE)

$$\begin{array}{c}
 \frac{p \not\vdash i \quad p \not\vdash i'}{p \not\vdash (i \dot{\vee} i')} \text{NonFC } \dot{\vee} + \\
 \\
 \frac{p \not\vdash (i \dot{\vee} i')}{p \not\vdash i} \text{NonFC } \dot{\vee} -_l \quad \frac{p \not\vdash (i \dot{\vee} i')}{p \not\vdash i'} \text{NonFC } \dot{\vee} -_r
 \end{array}$$

NEGATION

TYPING

$$\frac{i : \text{Imp}}{\neg i : \text{Imp}}$$

SATISFACTION

$$\frac{p \triangleright i}{(\neg p) \triangleright (\dot{\neg} i)} \dot{\neg} + \quad \frac{(\neg p) \triangleright (\dot{\neg} i)}{p \triangleright i} \dot{\neg} -$$

“Not the case that (in the future) John comes to the party.” \triangleright *“Don’t come to the party [John]!”*

CONDITIONALS AND PSEUDO IMPERATIVES

Expressions that combine propositions with imperatives (modulo some restrictions).

- 1 *"If you see John, say hello."*
- 2 *"Have another blanket or you will be cold."*
- 3 *"Keep walking or I will shoot."*
- 4 *"Close the door and you will become warmer."*
- 5 *"Eat the nut and you will die."*

We can add rules that deal with these examples, including rules that allow expressions to act as imperatives (1) propositions (4, 5) (following Franke 2005) or both (2, 3).

A more "pragmatically aware" analysis may require the use of a preference logic for some of these examples.

SKETCH OF A MODEL

We can provide a model given in terms of (event-based) computer programs.

- Imperatives are external events.
- Agents are either threads or variables.
- Propositions are statements about states.
- Future tense refers to subsequent states.
- An imperative is satisfied if some future state is as requested by the imperative event.

The purpose of the model is to shown formal consistency of the proof rules, rather than providing a “realistic” model of agents and imperatives.

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TYPES OF NEGATION

ISSUE *“Don’t come to the party!”*

Does this mean *“Don’t be present at the party!”*, or *“Don’t take steps to be at the party!”*? (Hamblin).

OBSERVATION Isn’t this just a generic telic/atelic issue for negation?

SOLUTION *No specific, or ad hoc solution for imperatives is called for.*

NEGATIVE ACTIONS

ISSUE *“Don’t buy an apple!”*

Appears to require us to quantify over all actions of an agent and their consequences
(Status of “intent”? Not so clear).

OBSERVATION Something that has to be addressed by any formalisation expressed in terms of overt actions and events

SOLUTION *Satisfied if addressee can be described as not having bought an apple: thus abstracting away from the issue of characterising negative actions (avoiding/preventing something from coming about).*

ROSS' PARADOX

OUTLINE OF THE ISSUE

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“Post the letter”

a logic of satisfaction that mirrors conventional propositional logic will allow us to infer

“Post the letter or burn the letter!”

(i.e. $a \vdash (a \vee b)$) and not the other way around.

- This seems at odds with intuitions, which suggest the inference should just go the other way ($(a \vee b) \vdash a$) [Kamp 73].
- Solutions often appeal to conversational maxims.

ROSS' PARADOX

AN OBSERVATION

- Perhaps the relationship with plans leads to a confusion between “refinement” (or “satisfactoriness” [Kenny]);
- When seeking to comply with a request $(a \vee b)$ we may *refine* this to a .
We would not *refine* a request a to $(a \vee b)$
- But the *satisfaction conditions* of a will also satisfy $(a \vee b)$.
(Cf elimation \vee introduction rules.)
- In general, if X can be refined to Y ($X \geq Y$), then $Y \vdash X$.

ROSS' PARADOX

A SOLUTION

- *The formal theory proposed here makes clear that the semantic inference here is about formal satisfaction, not plan refinement, or “practical inference” as such.*
- *How an agent is to go about “satisfying” an imperative is a pragmatic issue, which may involve refinement and practical inference, but this is to be distinguished from satisfaction conditions.*
- *As in formal methods, if the refined version (a) is satisfied, then the the original, more general version ($a \vee b$) will also be satisfied, but not the other way around.*
- *There is no need to appeal to conversational implicatures in the semantics.*

REFINEMENT CALCULUS

Sample rules for the refinement of imperatives.

- $i \geq i$
- $(i \dot{\vee} j) \geq i$
- If p true, then $(p \dot{\rightarrow} i) \geq i$
- $i \geq (i \dot{\wedge} j)$ (cf. *strengthening*)

Incomplete analysis: also need to consider applicability conditions (cf. preconditions)

- Presuppositions of $p![a]^\downarrow$ are the preconditions for complying with $p![a]$.
- Disjuncts with distinct preconditions. . .

Refine the “action” or the imperative? (cf. program or specification).

Acting as if it is the refined imperative that is being complied with.

WEAK DISJUNCTION

ISSUE In weak disjunction, there is an “order” to do $(a \dot{\vee} b)$, but there is no free-choice, it is just not yet clear which it should be.

OBSERVATION Pragmatically, to “obey” such a request presumably means ensuring that you do nothing that can prevent you fulfilling either a or b .

SOLUTION *We adopt a scoping approach [Cf Kamp 73], which appears sympathetic to such a pragmatic interpretation.*

Again, there is no need to appeal to conversational implicatures in the semantics.

OTHER ISSUES

All of the following are major problems

- Issues of temporal ordering of conjunctive imperatives
- Permission revision
- Pragmatics in general
 - What do you do you intend to happen following an utterance
 - What do you do when you hear an utterance

NOT PROBLEMS FOR IMPERATIVES

... AS SUCH

- However these are generic problems for theories of meaning. They are not specific to imperatives.
- We are happy to abstract away from these problems when considering a core logical theory for the formal semantics of indicatives.
- Why not do the same for a core formal semantic theory of imperatives?

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THE APPROACH

- Concentrate on purely semantic behaviour of core imperatives
- Three (or four?) aspects to the semantic behaviour
 - 1 Satisfaction criteria
 - 2 Refinement and “practical reasoning”
 - 3 Validity
- Focussed on satisfaction criteria in this presentation
- Tried to avoid confusion between *satisfaction* entailments and *refinement* relationships.

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SUMMARY

FORMALISATION OF SATISFACTION CRITERIA

- A proof theoretic account of the satisfaction criteria for core imperatives has been outlined.
- The theory abstracts away from some difficult issues, such as actions and events, and avoids becoming bogged down in generic issues that are not specific to imperatives.
- It is formulated in a way that makes it (relatively) easier to determine whether we agree with particular patterns of inference.
- Some established problems concerning negation and disjunction have been discussed, and (hopefully) resolved.

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FUTURE WORK

SATISFACTION

- Quantification (any, some, every etc.)
- Relationship to deontic logic
- Logic of Satisfaction for deontic logic?

VALIDITY

- Formalise a theory of felicity for imperatives

REFINEMENT AND PRACTICAL REASONING

- Formalise a 'calculus of refinement' for imperatives
- Reconsider the notion of 'practical reasoning'
- Relate to a pragmatic analysis

The End