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# Imperatives: a judgemental analysis

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**Abstract.** This paper proposes a framework for formalising intuitions about the behaviour of imperative commands. It seeks to capture notions of satisfaction and coherence. Rules are proposed to express key aspects of the general logical behaviour of imperative constructions. A key objective is for the framework to allow patterns of behaviour to be described while avoiding making any commitments about how commands, and their satisfaction criteria, are to be interpreted. We consider the status of some conundrums of imperative logic in the context of this proposal.

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*Keywords:* imperatives, satisfaction, consistency, coherence, paradoxes

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## Introduction and overview

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The objective of this paper is to propose a framework for formalising intuitions about the behaviour of imperatives. The intention is for the framework to allow the unambiguous and succinct characterisation of behaviour without making a commitment to any specific reductive analysis.

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As a first approximation, we take imperatives to express commands that typically require some action, activity or state of affairs to be brought about, or avoided, for them to be deemed *satisfied*. A typical example is (1).

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(1) “Close the door!”

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Such an imperative may be judged to be satisfied if the door is closed by the intended recipient of the imperative.

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For the purposes of this paper, it is not necessary to know exactly which linguistic forms and interpretations are appropriately characterised as “imperatives”, or “commands”, nor exactly what constitutes “satisfaction” of a given imperative. All that is assumed is that there is a semantic analogue of an imperative, and that such imperative expressions have satisfaction criteria. Furthermore, some imperatives (or their satisfaction criteria) may be judged to be inconsistent with each other. Here we are implicitly assuming a semantic notion of an “imperative”; not all natural language expressions that

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30 are syntactically in imperative form need necessarily express an imperative  
31 command in the sense intended here.<sup>1</sup>

32 We will take *satisfaction* to play a role somewhat akin to that of *truth*  
33 in accounts of indicative statements. We will use a notion of *consistency* of  
34 imperatives, or their satisfaction criteria, to determine whether a ‘commanding  
35 authority’ is *incoherent*. These notions of consistency and coherence can be  
36 seen to impose constraints on what it is to be a *rational* authority.

37 A key objective is to allow intuitions about imperatives to be formu-  
38 lated and expressed as directly as possible, without assuming any particular  
39 interpretation, such as possible worlds, actions, or non-classical notions of  
40 entailment. The aim is for us to be able to consider imperatives in isolation,  
41 without being confounded and distracted by other, independent, foundational  
42 issues.

## 43 1. Imperatives in Natural Language

44 Imperatives can be combined with each other and with indicatives. Here we  
45 provide some examples. These examples are not intended to be exhaustive.  
46 They merely highlight some key aspects of the behaviour of imperatives that  
47 we seek to capture (§2). Some of these linguistic constructions will also feature  
48 in the discussion of various conundrums and paradoxes (§3).

### 49 1.1. Conjunction

50 Imperatives can be conjoined with each other.

51 (2) “*Jump out of the window, and land on the pile of mattresses!*”

52 In general, we should avoid assuming this is equivalent to commanding the  
53 individual conjuncts separately. For example, we should not assume that  
54 (2) entails the command “*Jump out of the window!*”. This can be seen  
55 by considering the case where partial satisfaction is explicitly stated to be  
56 undesirable, as in “. . . *But don’t just jump out of the window, . . .!*” [23].

57 It would be incoherent to command (2) while also commanding (3).

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<sup>1</sup>Arguably, it may be better to take “imperative” to refer to the linguistic clause type (which typically, although not always, express some form of command), and use “command” or similar for the semantic notion (which typically, although not always, may be expressed by an imperative). But this may also be confusing in those cases where an imperative clause does not contribute something that is best interpreted as a command that is intended to be satisfied. Regardless of nuances in terminology, the question being pursued here is how to formalise the canonical interpretation of imperatives within a logical framework.

58 (3) “Don’t jump out of the window!”

59 The case made here is that such a notion of *(in)coherence* can stand as a  
60 weak proxy for *validity*. Instead of saying that there is a valid entailment  
61 from command  $a \wedge b$  to command  $b$ , for example, we can argue that it is  
62 *incoherent* for an authority to command both  $a \wedge b$  and  $\neg b$  at the same time,  
63 as their satisfaction criteria are inconsistent with each other (§2.2.3).

64 That imperative force fails to distribute to the conjuncts in (2) is perhaps  
65 due to a sequential interpretation of conjunction, where “*and*” is interpreted  
66 as “*and then*”. An analysis of sequential commands is offered in §2.4.

## 67 1.2. Free-Choice Disjunction

68 Disjunctive imperatives (4) often give rise to a free choice as to how they are  
69 to be satisfied [25].

70 (4) “Go to the beach or play in the park!”

71 With free-choice disjunction there is a sense in which “permission” is granted  
72 to do either, by indicating a space of legitimate possibilities [10]. In this  
73 regard it appears incoherent to combine the command (4) with (5), unless  
74 the latter is taken to be a correction, or implicit refinement.

75 (5) “Don’t go to the park!”

76 Given this permissive, free-choice interpretation of disjunction, it seems  
77 we should not be able to “introduce” a disjunctive command (cf. §3.1). That  
78 is (4) should not follow from the command (6), even though the satisfaction  
79 conditions of the latter should also satisfy the former.

80 (6) “Play in the park!”

81 There is a question as to whether free-choice disjunction supports an  
82 *exclusive* or *inclusive* interpretation. Under an exclusive interpretation of (4),  
83 going to the beach *and* playing in the park is not permissible and would fail  
84 to satisfy the command. We will formulate both weak (inclusive) and strong  
85 (exclusive) interpretations (§2.2.1).

86 Free-choice disjunction arises in other contexts. Ideally we might wish to  
87 obtain a general solution to this issue [42], rather than one that is specific to  
88 imperatives.

89 **1.3. Weak Disjunction**

90 An alternative interpretation of disjunctive examples such as (4) is where the  
 91 subject has been commanded to go to the beach or to play in the park, but  
 92 it is underspecified as to which is the case [26]. This is sometimes referred  
 93 to as *weak disjunction*. It might be analysed by some form of meta-level  
 94 disjunction. This paper focuses on the free-choice interpretation.

95 **1.4. Negation**

96 Commands can contain negation, as in (7).

97 (7) *“Don’t go to the beach!”*

98 It is incoherent to command something and its negation. Indeed it is incoher-  
 99 ent to endorse any commands with satisfaction criteria that are inconsistent  
 100 (§2.2.3).

101 These cases are to be distinguished from the ‘meta-level’ negation of (8),  
 102 where the existence of a command is being denied.

103 (8) (a) *“It is not the case that you are commanded to go to the beach!”*

104 (b) ‘There is no command of the form *“Go to the beach!”*’

105 This paper focuses on object-level negation.<sup>2</sup>

106 **1.5. Conditionals**

107 Conditionals may be formed where the antecedent is a proposition, and the  
 108 consequent is an imperative (9).

109 (9) *“If you see John, say hello!”*

110 If the antecedent is true, then the satisfaction conditions of the consequent  
 111 become salient.

112 We may wonder whether we can infer that *“say hello!”* has actually been  
 113 commanded *as such* in the event that John is seen<sup>3</sup> or whether the conditional  
 114 command is effectively irreducible (cf. §2.2.1).

115 It is conceivable that *avoiding John* counts as satisfaction of (9). This  
 116 interpretation is perhaps more salient in cases where the antecedent is morally  
 117 questionable, as with (10).<sup>4</sup>

118 (10) *“If you hit John, then apologise!”*

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<sup>2</sup>The question of “wide scope” negation is discussed by Han [17].

<sup>3</sup>This is known as “propositional detachment”.

<sup>4</sup>Such cases might be expressed more naturally using modal propositions, as in *“If you hit John, then you must apologise!”*. There is discussion of conditionals by Charlow [8] and

119 **1.6. Pseudo Imperatives**

120 Pseudo-imperatives are another form of expression in which imperatives are  
 121 combined with propositions [9, 13, 12, 28, 40, 49, 43]. Syntactically we can  
 122 consider two variants, one involving disjunction, the other conjunction.

123 **1.6.1. Pseudo-Or**

124 Disjunctive pseudo-imperatives are expressions such as (11).

125 (11) *“Take another drink, or you will be thirsty!”*

126 This appears to have imperative force—namely *“Take another drink!”*—  
 127 combined with propositional information—*“If you fail to take another drink,  
 128 you will be thirsty”* (cf. [13]). This might be considered a variant of free-choice  
 129 disjunction, where the second disjunct will come about “by default” if the  
 130 first disjunct is not satisfied. We may question whether the propositional  
 131 interpretation is effectively offering any kind of *guarantee* that the proposi-  
 132 tional component will be false if the imperative component is satisfied. It  
 133 would appear somewhat incoherent to say (11) together with (12) (§2.2.3).

134 (12) *“Don’t take another drink!”*

135 **1.6.2. Pseudo-And**

136 Conjunctive pseudo-imperatives appear to come in two flavours, exemplified  
 137 by (13a) and (13b).

138 (13) (a) *“Take another step and you will die!”*

139 (b) *“Take another step and you will see the treasure!”*

140 These can be taken to correspond to a threat, or a promise, respectively [27].<sup>5</sup>

141 Arguments have been made that conjunctive pseudo-imperatives are  
 142 essentially propositional in nature, and are not imperatives as such. They  
 143 are judged to be true propositions if the propositional conjunct is true when  
 144 the “imperative” conjunct is satisfied [13]. Determining what counts as a

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by Kaufmann and Schwager [45]. The view that avoidance is acceptable is supported by Schwager [42].

<sup>5</sup>Pseudo-conjunctions may also be used to express a quandary rather than a threat or promise [42]. For example: “(context: What should you say if someone comes from the state television company and asks if you own a television set?) Say no and he’ll go away for a while. Say yes and he will order you to pay. Over and over again.” (example from <http://www.thejapanfaq.net>, provided by an anonymous reviewer).

145 promise or a threat requires a value judgement, either of the outcome itself,  
 146 or some evaluation of the outcome weighed against the cost of satisfying the  
 147 imperative clause. We will not consider such pragmatic issues here.<sup>6</sup>

148 As with the disjunctive case, we may question other aspects of the logical  
 149 behaviour of such expressions, in this case whether it behaves like material  
 150 implication, or whether it is better characterised in some other way—as a  
 151 “causative” or “hypothetical” conditional, for example.<sup>7</sup>

## 152 2. Formalisation

153 The formalisation that follows is intended to be used as a framework for  
 154 expressing theories about the semantics of imperatives. Where possible, only  
 155 minimal ontological commitments are made. For example, imperatives will  
 156 not be required to have, or be related to, overtly propositional content, and  
 157 their satisfaction criteria will not be tied to any particular notion, such as  
 158 the post-conditions of actions. That is not to say that the content of an  
 159 imperative cannot be characterised as relating to some agentive sentence [7],  
 160 or that their satisfaction cannot be expressed in terms of actions, merely that  
 161 no such commitment is made here.

162 First we give the syntax and notation, and then rules for the judgements  
 163 of inconsistency, incoherence, satisfaction, and truth.<sup>8</sup> An abbreviated no-  
 164 tation for systems-of-commands is used in the analysis of inconsistency and  
 165 incoherence. A notion of an “obedient subject” is also discussed.

### 166 2.1. Syntax

167 To formalise the interpretation of imperatives and propositions, and express  
 168 judgements about them, we must have a syntax for their representation.  
 169 Given that imperatives may be combined with propositions, there is some  
 170 interplay between these two categories.

171 After the object level syntax has been presented, the notation for ex-  
 172 pressing judgements about members of the categories of imperatives and

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<sup>6</sup>Some claim that there are two distinct analyses for the conjunctive case, one as conditional proposition, for cases like (13a), the other as an imperative, perhaps combined with a conditional for cases like (13b) [43].

<sup>7</sup>One issue that needs to be considered is whether pseudo imperatives are embeddable in arbitrary propositional contexts. It appears that this may not be the case [49].

<sup>8</sup>For compactness, the syntax is given in BNF notation. An alternative would be to present the entire theory, including the syntax, in terms of judgements, as with Typed Predicate Logic [47].

173 propositions can be given. This language of judgements is used to express  
 174 rules governing the behaviour of imperatives and propositions (§2.2). To  
 175 improve the readability of some of these rules, abbreviations for systems of  
 176 commands are introduced.

### 177 2.1.1. Imperatives

178 We assume there is a category of expressions  $i$  that represents the substantive  
 179 content of atomic commands, sometimes known as *practives*. More complex  
 180 imperatives can then be formed from these atomic commands.

181 (14) *Basic imperatives*

$$182 \quad I_b ::= i \mid \neg I_b \mid I_b \wedge I_b \mid I_b \wedge_T I_b \mid I_b \vee_{FC} I_b$$

183 Such practives may be distinct from propositions [7]. Where possible, the  
 184 formalisation will remain neutral on such ontological issues.

185 The operator  $\vee_{FC}$  is used to highlight the intended free-choice interpreta-  
 186 tion, although it may be appropriate just to use  $\vee$  (especially if we take the  
 187 view that weak disjunctive commands are not to be expressed with a narrow  
 188 scope disjunction).

189 The operator  $\wedge_T$  is used for the sequential interpretation of conjunctions  
 190 as “*and then*”, although it may be appropriate just to use  $\wedge$ , and adopt a  
 191 more general approach to the analysis of sequential conjunction.

192 The category of conditional imperatives can be given as in (15).

193 (15) *Conditionals*

$$194 \quad I_c ::= P_c \rightarrow I_b \mid P_c \rightarrow I_c$$

195 where  $P_c$  is a classical proposition (17).

196 This syntax assumes that the conditional structure can nest, provided only  
 197 the right-most, final consequent is a basic imperative.

198 The category that represents the content of all imperatives, including  
 199 basic, conditional, and disjunctive pseudo-imperatives is given by (16).

200 (16) *Imperatives*

$$201 \quad I ::= I_b \mid I_c \mid I_b \vee P_c$$

202 where  $P_c$  is a classical proposition (17).

203 We will typically use “ $a$ ” to denote an imperative when discussing judge-  
 204 ments about imperatives or imperative constituents.

### 2.1.2. Propositions

Classical propositions can have their usual representation.

(17) *Classical Propositions*

$$P_c ::= p_c \mid \neg P_c \mid P_c \wedge P_c \mid P_c \vee P_c \mid P_c \rightarrow P_c$$

where  $p_c$  represents atomic classical propositions.

The category of classical propositions can be extended to a more general category that includes the propositional characterisation of pseudo-imperative expressions.

(18) *Generalised Propositions*

$$P ::= P_c \mid I_b \wedge P \mid I_b \vee P_c$$

Typically, we will use “ $p$ ” to denote an individual proposition when discussing judgements involving propositions, or propositional constituents.<sup>9</sup>

### 2.1.3. Judgements

A critical part of the proposed framework is a collection of *judgements* that can be made about imperatives, and propositions. Patterns of entailment can be formulated using these judgements. In the case of propositions ( $P$ ), we have judgements corresponding to whether they are true or false. In the case of individual imperatives ( $I$ ), there are judgements as to whether they have been commanded, satisfied, or not satisfied. Given a collection of imperatives ( $I, \dots, I$ ), there are judgements as to whether they, or their satisfaction criteria, are inconsistent, and whether the agent responsible for commanding them is incoherent.

(19) *Judgements*

$$J ::= P \text{ True} \mid P \text{ False} \mid I \text{ Commanded}_\alpha \mid I \text{ Satisfied}_\sigma \mid I \text{ unSatisfied}_\sigma \\ \mid (I, \dots, I) \text{ Inconsistent} \mid \alpha \text{ Incoherent}$$

Some of the judgements have the subscript  $\alpha$ , to indicate the relevant authority, and  $\sigma$ , to represent the subject. Although technically redundant in the current presentation, this notation can help clarify the intended agent when it comes to commanding and satisfying an imperative.<sup>10</sup>

<sup>9</sup>The formalisation given here excludes the embedding of pseudo-imperatives within propositional contexts [49].

<sup>10</sup>Where relevant, it can be assumed that the authority  $\alpha$  is expecting subject  $\sigma$  to comply. We do not consider whether imperatives may have overt subjects that differ from  $\sigma$  [41, 42, 51].

233 The judgement  $p$  True ( $p$  False) mean that  $p$  is judged to be true (false,  
 234 respectively). For complex propositions, we will assume that such judgements  
 235 behave in a way that corresponds to a classical logic for propositions.

236 The judgement “ $a$  Commanded $_{\alpha}$ ” means that authority  $\alpha$  has issued the  
 237 command  $a$ , where  $a$  is the semantic analogue of an imperative. The content  
 238 of the commands is assumed to be highly ‘inscriptional’ in nature: even if  
 239  $b$  True follows from  $a$  True, it does not mean that  $b$  Commanded $_{\alpha}$  necessarily  
 240 follows from  $a$  Commanded $_{\alpha}$ .

241 The judgement “ $a$  Satisfied $_{\sigma}$ ” means that subject  $\sigma$  has satisfied the (pu-  
 242 tative) command  $a$ . We assume that  $a$  Satisfied $_{\sigma}$  (and indeed  $a$  unSatisfied $_{\sigma}$ )  
 243 does *not* imply or presuppose  $a$  Commanded $_{\alpha}$ . This allows us to consider  
 244 entailments between satisfaction conditions without giving rise to any inap-  
 245 propriate entailments concerning what has actually been commanded.<sup>11</sup>

246 The judgement “ $a$  unSatisfied $_{\sigma}$ ” means that subject  $\sigma$  has *not* satisfied the  
 247 (putative) command  $a$ . We aim to be neutral as to whether “ $a$  Satisfied $_{\sigma}$ ” and  
 248 “ $a$  unSatisfied $_{\sigma}$ ” are *contradictory* as opposed to being merely *contrary*. Being  
 249 contrary, then it should not be possible to maintain both “ $a$  Satisfied $_{\sigma}$ ” and  
 250 “ $a$  unSatisfied $_{\sigma}$ ” simultaneously (23). If they were also contradictory, then all  
 251 imperatives would have to be satisfied, or not satisfied. It can be argued that  
 252 this is not the necessarily the case. The command (20) is clearly satisfied if  
 253 the bill is paid within the twenty-one days. It is unsatisfied if no payment is  
 254 made by the end of the twenty-one days.

255 (20) “*Pay this bill within twenty-one days!*”

256 But within the twenty-one days, while the bill remains unpaid, we might  
 257 wish to maintain that (20) is not (yet) “satisfied” nor “unsatisfied”. There  
 258 is perhaps a debate to be had about the most appropriate terminology to  
 259 describe such notions unambiguously.

260 The judgement “ $a_1, \dots, a_n$  Inconsistent” indicates that the imperatives  
 261 are inconsistent. The intuition is that if  $a_1, \dots, a_n$  were translated into  
 262 propositions, either directly or by way of their satisfaction criteria, they  
 263 would be inconsistent with each other, in the sense that if the corresponding  
 264 propositions were all true together, they would allow the derivation of any  
 265 proposition.

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<sup>11</sup>Given that  $a$  Satisfied $_{\sigma}$  does not presuppose there was a command  $a$  Commanded $_{\alpha}$ , it follows that  $a$  Satisfied $_{\sigma}$  cannot mean that a command was *intentionally* satisfied: there may have been no such command. Satisfaction is then an ‘extensional’ notion, that can be contrasted with the intensional, or inscriptional, flavour of  $a$  Commanded $_{\alpha}$ . Additional machinery would be required if it were necessary to distinguish between intentional and incidental satisfaction of a command.

266 The final judgement, “ $\alpha$  Incoherent”, is used to indicate that authority  $\alpha$   
 267 seeks to impose inconsistent commands. We assume that a rational authority  
 268 would seek to avoid issuing commands that give rise to a judgement of  
 269 incoherence. But the logic should be able to cope with an incoherent authority.

270 **A Reduction** We could try to reduce *satisfaction* of an imperative  $a$  to  
 271 *truth* of a propositional analogue  $a'$  of that imperative, with an implicit  
 272 subject  $\sigma$ . In particular,  $a$  Satisfied $_{\sigma}$  could be reduced to  $a'_{\sigma}$  True. We  
 273 could go further, and have imperatives  $a$  belong directly to the category  
 274 of propositions, making  $a$  Satisfied $_{\sigma}$  a notational variant of  $a_{\sigma}$  True. Both  
 275 of these moves will be avoided in the current account in order to leave  
 276 open the possibility of alternative notions of satisfaction, for example where  
 277 satisfaction is characterised directly by actions rather than propositions. It  
 278 also allows us to maintain a clear ontological distinction between imperatives  
 279 and propositions.<sup>12</sup>

#### 280 2.1.4. Abbreviation for Systems of Commands

281 It is helpful to have a notation for representing a system of commands, not  
 282 just individual commands. This can be used to represent a context in which  
 283 a collection of commands are to be considered together, as in §2.2.3. For  
 284 this purpose we will use  $\Sigma$  to refer to the collection of commands  $a_1, \dots, a_n$ ,  
 285  $\Sigma$  Commanded $_{\alpha}$  to represent the judgement that all these commands have  
 286 indeed been commanded, and  $\Sigma$  Satisfied $_{\sigma}$  to represent the judgement that  
 287 they have been satisfied (21).

- 288 (21) (a) “ $\Sigma$ ” stands for “ $a_1, \dots, a_n$ ”.  
 289 (b) “ $\Sigma$  Commanded $_{\alpha}$ ” stands for “ $a_1$  Commanded $_{\alpha}, \dots, a_n$  Commanded $_{\alpha}$ ”.  
 290 (c) “ $\Sigma$  Satisfied $_{\sigma}$ ” stands for “ $a_1$  Satisfied $_{\sigma}, \dots, a_n$  Satisfied $_{\sigma}$ ”.

291 No temporal ordering or precedence is intended when we write  $a_1, \dots, a_n$ .

## 292 2.2. Rules

293 Here, core patterns of behaviour of imperatives are expressed using rules of  
 294 the form (22) over judgements  $J$  (19).

$$295 \quad (22) \quad \frac{J_1 \quad \dots \quad J_n}{J}$$

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<sup>12</sup>This is not to say that no reduction from imperatives to propositions is possible.

296 Essentially, (22) expresses the claim that judgement  $J$  follows from the  
 297 judgements  $J_1, \dots, J_n$ .<sup>13</sup>

298 In some cases we initially give rules that characterise a generally un-  
 299 controversial core behaviour, followed by stronger rules that may be more  
 300 contentious.

### 301 2.2.1. Satisfaction

302 We use  $a \text{ Satisfied}_\sigma$  and  $a \text{ unSatisfied}_\sigma$  to express the judgements that impera-  
 303 tive  $a$  has been satisfied, or not, by subject  $\sigma$ . As previously noted (cf. §2.1.3),  
 304 we do not seek to reduce notion of satisfaction to something else, such as a  
 305 propositional description of a state, or an action. All that is required is for  
 306 there to be such a notion for every imperative, even if the satisfaction criteria  
 307 are not actually realisable in some cases.

308 It would be inconsistent to assert that the same imperative had both been  
 309 satisfied and not satisfied.<sup>14</sup>

$$310 \quad (23) \quad \frac{a \text{ Satisfied}_\sigma \quad a \text{ unSatisfied}_\sigma}{\perp}$$

311 While  $a \text{ Satisfied}_\sigma$  and  $a \text{ unSatisfied}_\sigma$  are contrary, we wish to avoid  
 312 requiring that they be contradictory. This means that it is sometimes necessary  
 313 to formulate rules for both the positive and negative cases explicitly, as in (24).

314 As mentioned before (2.1.3), here judgements of the form  $a \text{ Satisfied}_\sigma$ ,  
 315 and  $a \text{ unSatisfied}_\sigma$ , do *not* presuppose or imply  $a \text{ Commanded}_\alpha$ .

316 **Conjunction** Conjunction is subject to the expected rules for satisfaction.  
 317 Both conjuncts must be satisfied for their conjunction to be satisfied. The  
 318 conjunction is judged to be unsatisfied if either conjunct is not satisfied.

319 (24) *Conjunction*

$$320 \quad (a) \quad \frac{a \text{ Satisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{(a \wedge b) \text{ Satisfied}_\sigma}$$

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<sup>13</sup>These rules can be thought of as being akin to axioms of the form  $J_1 \dots J_n \rightarrow J$  in the meta-language. While the rules provide introduction and elimination rules for each connective, here we do not consider the question of local soundness, completeness and the existence of normal forms. The primary objective here is to illustrate an approach to formalising the behaviour of imperatives, rather than explore the formal properties of a particular characterisation.

<sup>14</sup>For a specific theory of satisfaction it is appropriate to ensure that there are no counter-intuitive results, including those that lead to formal inconsistency of judgements, and the derivation of  $\perp$ . We do not do so here; the proposed rules are intended only to exemplify the general approach (cf. §2.5).

$$\begin{array}{l}
321 \quad (b) \frac{a \text{ unSatisfied}_\sigma}{(a \wedge b) \text{ unSatisfied}_\sigma} \quad (c) \frac{b \text{ unSatisfied}_\sigma}{(a \wedge b) \text{ unSatisfied}_\sigma} \\
322 \quad (d) \frac{(a \wedge b) \text{ Satisfied}_\sigma}{a \text{ Satisfied}_\sigma} \quad (e) \frac{(a \wedge b) \text{ Satisfied}_\sigma}{b \text{ Satisfied}_\sigma} \\
323 \quad (f) \frac{(a \wedge b) \text{ unSatisfied}_\sigma \quad a \text{ Satisfied}_\sigma}{b \text{ unSatisfied}_\sigma} \\
324 \quad (g) \frac{(a \wedge b) \text{ unSatisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{a \text{ unSatisfied}_\sigma}
\end{array}$$

325 Sequential “and then” conjunction is considered in §2.4.

326 **Free Choice** The core behaviour of free-choice disjunction is given by the  
327 rules in (25), where the disjunction is satisfied if either one of the disjunctions  
328 is satisfied (and the other is not), and is not satisfied if both are not satisfied.

329 (25) *Basic Free Choice*

$$\begin{array}{l}
330 \quad (a) \frac{a \text{ Satisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \quad (b) \frac{a \text{ unSatisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \\
331 \quad (c) \frac{a \text{ unSatisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{(a \vee_{FC} b) \text{ unSatisfied}_\sigma} \\
332 \quad (d) \frac{(a \vee_{FC} b) \text{ Satisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{a \text{ Satisfied}_\sigma} \\
333 \quad (e) \frac{(a \vee_{FC} b) \text{ Satisfied}_\sigma \quad a \text{ unSatisfied}_\sigma}{b \text{ Satisfied}_\sigma} \\
334 \quad (f) \frac{(a \vee_{FC} b) \text{ unSatisfied}_\sigma}{a \text{ unSatisfied}_\sigma} \quad (g) \frac{(a \vee_{FC} b) \text{ unSatisfied}_\sigma}{b \text{ unSatisfied}_\sigma}
\end{array}$$

335 We can strengthen this core behaviour by adopting an exclusive inter-  
336 pretation of free-choice, where satisfying both disjuncts leads to an explicit  
337 failure to satisfy the free-choice imperative.

338 (26) *Explicitly Exclusive Free Choice*

$$\begin{array}{l}
339 \quad (a) \frac{a \text{ Satisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \quad (b) \frac{a \text{ unSatisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \\
340 \quad (c) \frac{a \text{ unSatisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{(a \vee_{FC} b) \text{ unSatisfied}_\sigma} \quad (d) \frac{a \text{ Satisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{(a \vee_{FC} b) \text{ unSatisfied}_\sigma} \\
341 \quad (f) \frac{(a \vee_{FC} b) \text{ unSatisfied}_\sigma \quad b \text{ Satisfied}_\sigma}{a \text{ Satisfied}_\sigma} \\
342 \quad (g) \frac{(a \vee_{FC} b) \text{ unSatisfied}_\sigma \quad a \text{ Satisfied}_\sigma}{b \text{ Satisfied}_\sigma}
\end{array}$$

343 This captures the intuition that both *going to the beach* and *playing in the*  
 344 *park* would not satisfy the exclusive interpretation of (4) “*Go to the beach or*  
 345 *play in the park!*”.

346 In contrast, an inclusive free-choice is supported explicitly by (27).<sup>15</sup>

347 (27) *Explicitly Inclusive Free Choice*

$$\begin{array}{l}
 348 \quad (a) \frac{a \text{ Satisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \quad (b) \frac{b \text{ Satisfied}_\sigma}{(a \vee_{FC} b) \text{ Satisfied}_\sigma} \\
 349 \quad (c) \frac{a \text{ unSatisfied}_\sigma \quad b \text{ unSatisfied}_\sigma}{(a \vee_{FC} b) \text{ unSatisfied}_\sigma}
 \end{array}$$

350 **Negation** The judgements of  $a \text{ Satisfied}_\sigma$  and  $a \text{ unSatisfied}_\sigma$  are exclusive.

351 (28) *Negation*

$$\begin{array}{l}
 352 \quad (a) \frac{a \text{ Satisfied}_\sigma}{(\neg a) \text{ unSatisfied}_\sigma} \quad (b) \frac{a \text{ unSatisfied}_\sigma}{(\neg a) \text{ Satisfied}_\sigma} \\
 353 \quad (c) \frac{(\neg a) \text{ Satisfied}_\sigma}{a \text{ unSatisfied}_\sigma} \quad (d) \frac{(\neg a) \text{ unSatisfied}_\sigma}{a \text{ Satisfied}_\sigma}
 \end{array}$$

354 This does not mean that satisfaction is bivalent; there may be satisfaction  
 355 gaps. We could have a double negation rule, so  $(\neg \neg a) \text{ Satisfied}_\sigma$  if and only  
 356 if  $a \text{ Satisfied}_\sigma$  (similarly for  $(\neg \neg a) \text{ unSatisfied}_\sigma$ ).

357 **Conditionals** Initially we give a very weak analysis of conditional impera-  
 358 tives. As conditionals have propositional content, the rules that govern them  
 359 involve judgements of truth, in addition to satisfaction.

360 (29) *Conditionals*

$$\begin{array}{l}
 361 \quad (a) \frac{p \text{ True} \quad a \text{ Satisfied}_\sigma}{(p \rightarrow a) \text{ Satisfied}_\sigma} \quad (b) \frac{p \text{ True} \quad a \text{ unSatisfied}_\sigma}{(p \rightarrow a) \text{ unSatisfied}_\sigma} \\
 362 \quad (c) \frac{p \text{ True} \quad (p \rightarrow a) \text{ Satisfied}_\sigma}{a \text{ Satisfied}_\sigma} \quad (d) \frac{p \text{ True} \quad (p \rightarrow a) \text{ unSatisfied}_\sigma}{a \text{ unSatisfied}_\sigma}
 \end{array}$$

363 We may wonder whether preventing the antecedent  $p$  from becoming true  
 364 may count as satisfaction of the imperative. Such a view would justify (30).

365 (30) *Indirect Satisfaction of Conditionals*<sup>16</sup>

$$366 \quad (a) \frac{p \text{ False}}{(p \rightarrow a) \text{ Satisfied}_\sigma} \quad (b) \frac{(p \rightarrow a) \text{ Satisfied}_\sigma \quad a \text{ unSatisfied}_\sigma}{p \text{ False}}$$

<sup>15</sup>There are cases of free choice permission where the inclusive reading appears natural [5].

<sup>16</sup>Arguably, the second of these rules falls under the remit of §2.2.2, which considers truth judgements.

367 In some cases it might seem perverse, but consider (10) “*If you hit John,*  
 368 *then apologise!*”, which might be interpreted as an indirect command to avoid  
 369 hitting John.

370 **Contra-positives** Further strengthenings, and extensions, may be sug-  
 371 gested by considering *contra-positives*. In classical logic  $p \rightarrow q$  is equivalent  
 372 to its contra-positive  $\neg q \rightarrow \neg p$ . In the case of imperatives, we might want  
 373 to reflect on (31).<sup>17</sup>

- 374 (31) (a) “*If it is not daytime, don’t turn out the light*”  
 375 (b) “*Turn out the light only if it is daytime*”

376 These support the view that it may be appropriate to allow contra-positive  
 377 forms, so  $I_c$  includes  $I_b \rightarrow P_c$  (15) with satisfaction conditions that support  
 378 (32).

- 379 (32)  $(a \rightarrow p)$  Satisfied $_{\sigma}$  iff  $(\neg p \rightarrow \neg a)$  Satisfied $_{\sigma}$

380 We can argue that for conditional imperatives there is another notion of  
 381 contra-positive with respect to satisfaction, as follows.

- 382 (33)  $(p \rightarrow a)$  Satisfied $_{\sigma}$  if either  
 383 (a) if  $p$  True then  $a$  Satisfied $_{\sigma}$   
 384 (b) if  $a$  unSatisfied $_{\sigma}$  then  $p$  False.

385 Here, (33b) has the form of the contra-positive of (33a). The first disjunct  
 386 (33a) is already captured by (29a) and (33b) is captured by (30b).

387 **Pseudo-Or** Disjunctive pseudo-imperatives have the same satisfaction cri-  
 388 teria as their imperative constituent.

- 389 (34) *Pseudo-Or*  
 390 (a)  $\frac{a \text{ Satisfied}_{\sigma}}{(a \vee p) \text{ Satisfied}_{\sigma}}$  (b)  $\frac{a \text{ unSatisfied}_{\sigma}}{(a \vee p) \text{ unSatisfied}_{\sigma}}$   
 391 (c)  $\frac{(a \vee p) \text{ Satisfied}_{\sigma}}{a \text{ Satisfied}_{\sigma}}$  (d)  $\frac{(a \vee p) \text{ unSatisfied}_{\sigma}}{a \text{ unSatisfied}_{\sigma}}$

---

<sup>17</sup>Example (31) is based on an example suggested by an anonymous reviewer. We may wish to reflect on the extent to which it is possible to represent *permission* by way of contra-positive and free-choice imperatives.

### 2.2.2. Truth

Finally we can consider the judgements of truth. Such judgements are required for the analysis of pseudo-imperatives and conditional imperatives.<sup>18</sup>

(35) *Standard Connectives*: As for classical logic.

(36) *Pseudo-And*

$$(a) \frac{(a \wedge p) \text{ True} \quad a \text{ Satisfied}_\sigma}{p \text{ True}}$$

$$(b) \frac{a \text{ Satisfied}_\sigma \quad p \text{ True}}{(a \wedge p) \text{ True}} \quad (c) \frac{a \text{ Satisfied}_\sigma \quad p \text{ False}}{(a \wedge p) \text{ False}}$$

(37) *Pseudo-Or*

$$(a) \frac{(a \vee p) \text{ True} \quad a \text{ unSatisfied}_\sigma}{p \text{ True}}$$

$$(b) \frac{p \text{ True}}{(a \vee p) \text{ True}} \quad (c) \frac{a \text{ Satisfied}_\sigma}{(a \vee p) \text{ True}}$$

Note that here we talk of a pseudo-imperative *being true* (or *being false*) rather than *being asserted* (or *commanded*).<sup>19</sup>

Conjunctive pseudo-imperatives do not make any claim about whether the imperative component needs to be satisfied for the propositional conjunct to be true. From  $(a \wedge p) \text{ True}$  and  $a \text{ unSatisfied}_\sigma$  we would not wish to infer  $p \text{ False}$ .

As with propositional conditionals, we may still be interested in deciding what can and should be inferred in the case that the “antecedent” imperative conjunct is not satisfied. If it behaves like material implication, then we would have the following:

(38) *Strong Derivation of Pseudo-And*

$$\frac{a \text{ unSatisfied}_\sigma}{(a \wedge p) \text{ True}}$$

Pragmatically, it is not clear that such a pseudo-imperative would actually be asserted under these circumstances.

The initial rules for Pseudo-Or (34) do not say anything about the truth of propositional component in the event that the imperative component is

<sup>18</sup>Classical interpretations of conditional and disjunctive propositions are given here, although they do not necessarily provide the most appropriate foundation for the analysis of phenomena such as counter-factuals and free-choice disjunction.

<sup>19</sup>Pseudo-imperatives might be issued even if the relevant truth conditions are not satisfied. In such cases they could be characterised as “empty” threats or promises.

418 satisfied. To address this, we could have a “committed” version of Pseudo-Or  
 419 (39) for which the propositional outcome will be averted in the event that  
 420 the imperative is satisfied.

421 (39) *Committed Pseudo-Or*  
 422 
$$\frac{(a \vee p) \text{ True} \quad a \text{ Satisfied}_\sigma}{p \text{ False}}$$

423 In this case, the rules for Pseudo-Or introduction (37b,c) can be replaced by  
 424 those of (40).

425 (40) *Introduction of Committed Pseudo-Or*  
 426 (a)  $\frac{a \text{ Satisfied}_\sigma \quad p \text{ False}}{(a \vee p) \text{ True}}$       (b)  $\frac{a \text{ unSatisfied}_\sigma \quad p \text{ True}}{(a \vee p) \text{ True}}$   
 427 (c)  $\frac{a \text{ Satisfied}_\sigma \quad p \text{ True}}{(a \vee p) \text{ False}}$

428 It can be argued that rather than having a simple truth-conditional  
 429 meaning, pseudo-imperatives convey causative or counter-factual claims.  
 430 Such an analysis would require the adoption of an appropriate treatment of  
 431 counter-factual and causative statements, which lies outside the scope of the  
 432 current paper.<sup>20</sup>

### 433 2.2.3. Inconsistency and Incoherence

434 Here we present rules concerning judgements about the *consistency* of com-  
 435 mands and *coherence* authorities. These notions can be used to capture some  
 436 of the intuitions about incoherent combinations of commands. They can be  
 437 seen to correspond to a very weak form of validity. For example, even if we  
 438 wish to remain neutral as to whether it is right to infer  $a \text{ Commanded}_\alpha$  (as  
 439 such) follows from  $(a \wedge b) \text{ Commanded}_\alpha$  (cf. §1.1), we can still say that it is  
 440 *incoherent* of an authority to command  $\neg a$  at the same time as commanding  
 441  $(a \wedge b)$ .

442 In general, the satisfaction conditions of some putative commands can be  
 443 at odds with the satisfaction of others. It would be inconsistent for them to be  
 444 judged to be satisfied together. In such cases, the corresponding commands  
 445 would be inconsistent with each other, that is,  $(a_1, \dots, a_n) \text{ Inconsistent}$ , or  
 446  $\Sigma \text{ Inconsistent}$  (2.1.4). This is formulated in (41), using the abbreviations  
 447 given in §2.1.4.

<sup>20</sup>As already mentioned (footnote 7), the interpretation of conjunctive pseudo-imperative with a “positive” outcome as having genuinely imperative force [43] is ignored here; a case can be made this is best interpreted as a pragmatic effect.

$$(41) \frac{[\Sigma \text{ Satisfied}_\sigma]}{\perp} \\ \Sigma \text{ Inconsistent}$$

449 An alternative would be for the inconsistency of the commands themselves  
450 to be treated as a basic notion. It would then be inconsistent to claim such  
451 commands were all satisfied together (42).

$$(42) \frac{\Sigma \text{ Inconsistent} \quad \Sigma \text{ Satisfied}_\sigma}{\perp}$$

453 Whichever approach we take, there should be no *logical* difficulties in rep-  
454 resenting and making judgements about inconsistent collections of commands,  
455 *provided* that we avoid asserting that some mutually inconsistent commands  
456 are judged to have been satisfied.<sup>21</sup> It would however be incoherent for an  
457 authority to issue inconsistent commands (43).<sup>22</sup>

$$(43) \text{ Incoherent agents} \\ \frac{\Sigma \text{ Commanded}_\alpha \quad \Sigma \text{ Inconsistent}}{\alpha \text{ Incoherent}}$$

460 An authority will be judged incoherent for doing any of the following:

- 461 (a) issuing a conjunctive command in which the conjuncts are inconsistent.
- 462 (b) giving a free choice over things they have prohibited
- 463 (c) issuing a conditional command in which the imperative consequent is  
464 inconsistent with other commands, in the event the antecedent is true.
- 465 (d) issuing a disjunctive pseudo-imperative where the imperative constituent  
466 is inconsistent with other commands.

467 These follow as a consequence of the satisfaction criteria given in §2.2.1.

468 We may wish to strengthen the treatment of coherence of conditional  
469 imperatives so that a conditional ( $p \rightarrow a$ ) with a consequent  $a$  that is  
470 inconsistent with other commands is itself inconsistent regardless of the truth  
471 of the antecedent proposition  $p$ .

$$(44) \text{ Strong Consistency for Conditionals} \\ \frac{\Sigma \text{ Commanded}_\alpha \quad (p \rightarrow a) \text{ Commanded}_\alpha \quad (\Sigma, a) \text{ Inconsistent}}{\alpha \text{ Incoherent}}$$

<sup>21</sup>If  $a \text{ Satisfied}_\sigma$  were reduced to  $a_\sigma \text{ True}$  (§2.1.3), then this could be expressed directly in terms of classical consistency of propositions.

<sup>22</sup>Essentially (43), and (45), provide a connection between *commanding* and *satisfying*, in the former case by way of inconsistency (41).

474 This goes someway towards capturing the intuition that (9) “*If you see John,*  
 475 *say hello!*” would be inconsistent with a command not to say hello to anyone.  
 476 We might prefer to say that a commanding authority could only be judged  
 477 incoherent if  $p$  were *possible*.

### 478 2.3. Obedience

479 We may judge that some form of *transgression* ( $\mathcal{T}$ ) arises in the event that  
 480 authority  $\alpha$  has commanded something that subject  $\sigma$  has failed to satisfy  
 481 [2, 11, 50].

482 (45) *Transgression*

$$483 \frac{a \text{ Commanded}_\alpha \quad a \text{ unSatisfied}_\sigma}{\mathcal{T}_{\sigma,\alpha,a}}$$

484 Here the transgression  $\mathcal{T}$  is indexed with the subject, the authority, and  
 485 the command that has been transgressed. A more sophisticated analysis  
 486 would be required if it were necessary to distinguish between intentional and  
 487 co-incidental compliance, and the relationship between the subject  $\sigma$  and the  
 488 addressee of the imperative. If a system of authority is inconsistent, it may  
 489 not even be possible to comply. Here we do not consider the question of *when*  
 490 the satisfaction of a command is to be evaluated.

491 A subject  $\sigma$  who is obedient with respect to authority  $\alpha$  will seek to  
 492 minimise the number of transgressions, perhaps with a pragmatic value-  
 493 judgement in the case of an inconsistent authority, or conflicts between  
 494 authorities. Similar value-judgements could no doubt be employed to deter-  
 495 mine appropriate actions in the case of free-choice permission, and disjunctive  
 496 pseudo-imperatives.

497 This notion of a transgression might provide a suitable vehicle for a  
 498 pragmatic re-interpretation of validity with respect to formal and informal  
 499 specifications, as used in computer science for example. In effect, the account  
 500 proposed here gives specifications an intensional, or inscriptional, charac-  
 501 terisation which avoids a logical collapse when considering a specification  
 502 that contains inconsistencies [1, p123], whilst allowing partial fulfilment of a  
 503 specification.

### 504 2.4. Sequential Commands

505 Sequential commands [44] were alluded to in §1.1. A possible formulation  
 506 of the behaviour of imperatives of the form “*Do a and then do b!*” is given  
 507 in (46).

- 508 (46) (a) *Initial Coherence*  
509 
$$\frac{(a \wedge_T b) \text{ Commanded}_\alpha \quad \neg a \text{ Commanded}_\alpha}{\alpha \text{ Incoherent}}$$
- 510 (b) *Consequent Coherence (Strong)*  
511 
$$\frac{(a \wedge_T b) \text{ Commanded}_\alpha \quad \neg b \text{ Commanded}_\alpha}{\alpha \text{ Incoherent}}$$
- 512 (c) *Consequent Coherence (Weak)*  
513 
$$\frac{a \text{ Satisfied}_\sigma \quad (a \wedge_T b) \text{ Commanded}_\alpha \quad \neg b \text{ Commanded}_\alpha}{\alpha \text{ Incoherent}}$$
- 514 (d) *Satisfaction*  
515 (i) 
$$\frac{(a \wedge_T b) \text{ Satisfied}_\sigma}{a \text{ Satisfied}_\sigma} \quad \text{(ii) } \frac{(a \wedge_T b) \text{ Satisfied}_\sigma}{b \text{ Satisfied}_\sigma}$$
  
516 (iii) 
$$\frac{a \text{ Satisfied}_\sigma \text{ AND THEN } b \text{ Satisfied}_\sigma}{(a \wedge_T b) \text{ Satisfied}_\sigma}$$

517 This assumes some appropriate interpretation of “AND THEN” in the language  
518 of judgements.

519 A more refined approach could be to add a temporal dimension to systems  
520 of commands and their satisfaction, thus providing the means to formalise  
521 dynamic command systems.

## 522 2.5. Models for Imperative Theories

523 A model can be constructed in order to help demonstrate the consistency  
524 of any specific proposed collection of inference rules. In the case of the  
525 framework proposed here, one approach would be to model the propositions  
526  $P$  and imperatives  $I$ , and the operators that can combine them, as classes of  
527 terms in a combinatory calculus. Closure rules would then need to be given  
528 to reflect the syntax of  $P$  and  $I$ . Further classes and closure rules could then  
529 be added to model the judgements.

530 If appropriately constructed, the interpretation and the closure rules  
531 would demonstrate that there is a consistent interpretation of the proposed  
532 collection of inference rules. Producing a model-theoretic interpretation can  
533 help to demonstrate that a formal system is coherent. But, a model of this  
534 sort does not necessarily contribute directly to the *understanding* of the  
535 framework or the intuitions about the subject matter. Although important,  
536 here we view model-theory as playing a secondary, supporting role to the  
537 formal framework.<sup>23</sup>

<sup>23</sup>Other formal properties of the formal system also deserve analysis, including local sound-

538 **3. Imperative Dilemmas**

539 It is appropriate to discuss some of the dilemmas described in the literature  
 540 on imperatives, and demonstrate how they can be addressed within this  
 541 proposed framework.

542 **3.1. Ross’s Paradox**

543 By appealing to some seemingly contrary intuitions about the appropriate  
 544 rules for a logic of imperatives, Ross [38] argued that it is not possible to  
 545 formulate a logic for imperatives with inference rules that individually capture  
 546 both the notions of satisfaction and validity.

547 The core of Ross’s case can be illustrated by considering disjunction. If  
 548 we take classical logic to be the gold standard of *validity*, then given the  
 549 proposition  $p$  we should be able to infer  $(p \vee q)$ , where  $q$  is any proposition.  
 550 If this particular notion of validity is carried over directly into a “logic” of  
 551 imperatives, then if  $a$  is commanded, we can infer that  $(a \vee b)$  is commanded,  
 552 where  $b$  is any imperative. But, Ross argues, from (47) we probably do not  
 553 want to derive the command (48) as the satisfaction criteria of the latter  
 554 would licence the burning of the letter, which presumably the commanding  
 555 authority may consider undesirable.

556 (47) “*Post the letter!*”

557 (48) “*Post the letter or burn the letter!*”

558 This tension is resolved in the current account by maintaining a clear  
 559 distinction between judgements concerning *satisfaction* and those concerning  
 560 what has been *commanded*. Furthermore, the rules governing commands are  
 561 very weak. This allows the main thrust of Ross’s argument to be avoided:  
 562 we do not need to assume that notions of validity apply to the content of  
 563 commands. Classical validity can apply to judgements themselves without  
 564 necessarily applying directly to commands, or their content. If there is  
 565 a judgement that supports something closely resembling classical patterns  
 566 of inferences, it is that of *satisfaction*. Some problems are avoided if we  
 567 acknowledge that the judgement of satisfaction does not require or presuppose  
 568 that a command was issued.

---

ness, completeness and the existence of normal forms for derivations. Such investigations are beyond the scope of the current paper.

### 3.2. Free Choice

In the case of free-choice imperatives [3, 10, 25, 52] (§1.2), a key issue is that we appear to be “permitted” to comply with either command, but at the same time may only be allowed to comply with one. In the proposed account, the sense in which the free-choice imperative appears to licence both disjuncts is captured by the fact that it is incoherent to command something that contradicts (or whose satisfaction criteria contradicts) either of them. The exclusive interpretation is captured by ruling that the free-choice command is “satisfied” if *and only if* one disjunct is satisfied (26).<sup>24</sup>

### 3.3. Jørgensen’s Dilemma

The essence of Jørgensen’s Dilemma [24] is as follow: (a) logical inferences only hold between sentences with truth values; (b) imperatives have no truth values; therefore, (c) there should be no logical inferences between imperatives. And yet, (d) it still appears compelling to argue that imperatives do support some kind of logical entailment, as in (49). The dilemma is that the conclusion (c) and observation (d) appear to be at odds with one another.

- (49) (a)  $\frac{\text{“Keep your promises”} \quad \text{“This is a promise of yours”}}{\therefore \text{“Keep this promise.”}}$   
 (b)  $\frac{\text{“Love your neighbour as you love yourself”} \quad \text{“Love yourself”}}{\therefore \text{“Love your neighbour”}}$

The dilemma can be resolved if we take inferences involving propositions (50) to be short-hand for inferences over *truth judgements* about propositions (51).

- (50)  $\frac{p_1 \quad p_2 \quad \dots \quad p_n}{p}$   
 (51)  $\frac{p_1 \text{ True} \quad p_2 \text{ True} \quad \dots \quad p_n \text{ True}}{p \text{ True}}$

The claim (a) can then be generalised, and restated as “*logical inferences only hold between judgements*”. On this view, we *can* then have entailments involving expressions that do not have truth values—such as imperatives—provided that we identify the relevant judgements. For imperatives, the relevant judgements are those of *satisfaction* and *commanding* rather than *truth* [6]. In the current proposal, we seek to avoid one source of confusion by making explicit the intended nature of such judgements.

Even if otherwise satisfied by the current proposal’s resolution of Jørgensen’s Dilemma, the inquisitive reader may question what the proposal

<sup>24</sup>Free choice might also be analysed using resource sensitive logics [5] and to-do lists [36].

600 makes of the specific examples in (49). The theory as formalised in §2 does  
 601 not capture these entailments from general expressions to specific expression.  
 602 Even so, one can see that the arguments would appear to follow when couched  
 603 in terms of satisfaction. In the case of (49a) if the imperative is satisfied and  
 604 the proposition true in the premises, then the conclusion “*Keep this promise*”  
 605 is satisfied; if it is not, then either the propositional premise is false, or the  
 606 imperative premise is not satisfied. In the case (49b), if both premises are  
 607 satisfied, then the conclusion “*Love your neighbour*” is satisfied; if it is not,  
 608 then at least one of the premises cannot have been satisfied.

609 From the perspective of what has been *commanded*, the current account  
 610 would not allow us to infer that the commands in the conclusions of (49)  
 611 are judged to have been issued, although it would not be incoherent for an  
 612 authority to highlight the consequences of satisfying a command by issuing  
 613 the more specific imperatives.

#### 614 4. Related Work

615 Some analyses take imperatives to have to have an underlying “propositional  
 616 content” [19, 22, 24, 31, 46], or in terms of actions [18, 30].<sup>25</sup> In some  
 617 cases, the notion of an action is intimately related to a that of proposition  
 618 [30, 37, 44, 48]. For example, the proposition may be the *post-condition* of  
 619 the action. The imperative is then a request to perform this action in order  
 620 for the desired post-condition to become true.<sup>26</sup>

621 The current proposal avoids any direct reduction of imperatives to propo-  
 622 sitions, actions or other notions. Instead, it treats the semantic category of  
 623 imperatives as basic. Judgements about their satisfaction criteria are taken  
 624 to have the same status as judgements of truth in the case of propositions.  
 625 This notion of “satisfaction” corresponds to “fulfilment” criteria [6, 20, 29] or  
 626 “outcomes” [14].

627 The proposed framework abstracts away from any particular notion of  
 628 satisfaction, such as an action-based analysis. This allows core aspects of  
 629 the inferential behaviour of imperatives to be considered while avoiding  
 630 questions about actions, causality, events, intentions, the frame problem [33]  
 631 and the relationship between actions and events [4]. The framework can be

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<sup>25</sup>These are not the only options. For example, Mastop argues that one of the primary aspects of the meaning of imperatives is the notion of “acceptance” [32].

<sup>26</sup>It should be noted that actions, however formulated, need to take account of intent, not just post-conditions. A person who hangs a piece of bread in the water on a hook can only be described as engaging in the act of fishing if that is what she intends to do, regardless of whether or not she catches fish [16], or exactly how she goes about it.

632 enriched with actions and events if that is thought appropriate. Essentially  
633 the formalisation presented here can be considered as providing a normative  
634 structural characterisation that any more concrete model or implementation  
635 should satisfied.

636 It is common to adopt a *model-theoretic* methodology for semantic analysis.  
637 With such an approach, the primary task would be to provide an interpretation  
638 of imperatives in a model [30, 44]. Any representation language would then  
639 have only a secondary role. This can be contrasted with the current proposal,  
640 which seeks to provide a framework in which intuitions about inference  
641 behaviour are formulated more directly. An argument in favour of this  
642 approach is that it makes it easier for us to work at an appropriate level of  
643 abstraction. We can focus on the intuitions, rather than working around the  
644 technical difficulties that can arise when formulating a theory primarily as an  
645 “encoding” in some pre-existing model. The approach taken here also seems  
646 to make it easier to maintain a classical notion of inference, and avoid the  
647 need to adopt some form of defeasible entailment [3].

648 There are alternative formalisations that model imperatives in terms of  
649 “commitments” or “to do” lists [34, 35]. There is a sense in which these can be  
650 seen to be capturing the notion of satisfaction, and treating what has been  
651 commanded as something that need not be subject directly to any substantive  
652 rules of entailment.

653 There appear to be no other proposals that combine: (a) an explicit and  
654 unambiguous distinction between commanding and satisfying; (b) the avoid-  
655 ance of logical dilemmas in the face of incoherent commands; (c) consideration  
656 of conditional and pseudo imperatives; (d) a treatment of the permissive  
657 aspects of free-choice imperatives which does not resort to defeasible inference.

## 658 5. Conclusions and Future Work

659 The primary role of this contribution is to present a framework for formalising  
660 intuitions about the basic patterns of behaviour of judgements concerning  
661 imperatives. Some sample rules are proposed. The account could be extended  
662 to include quantification, discourse phenomena [30], pragmatic issues, and the  
663 relationship with deontic logic [15]. It may also be appropriate to find some  
664 way of unifying, or relating, those patterns of behaviour that are common to  
665 both *truth* and *satisfaction*. The formal properties of the system also merit  
666 exploration.

667 It might be argued that the current formalisation provides a mere *descrip-*  
668 *tion* of the patterns of behaviour of imperatives, and that only those accounts  
669 that seek to integrate an analysis of imperatives within some pre-existing

670 model can properly claim to count as a fully-fledged explanatory *theory*. Such  
 671 a claim may be seen to be reinforced if the appropriate patterns of behaviour  
 672 are obtained as a ‘natural’ consequence of some definitional reduction of  
 673 imperatives to other independently motivated notions.

674 A counter-argument is that the identification of appropriate ontological  
 675 categories and judgements, and the formalisation of patterns of behaviour  
 676 into precise rules, is an important and necessary step. Such an analysis can  
 677 be used to determine whether a proposed reduction preserves our intuitions.  
 678 This approach also allows us to work at an appropriate level of abstraction,  
 679 and avoids the risk of conflating intuitively distinct ontological categories.

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## 687 References

- 688 [1] ALCHOURRÓN, CARLOS E., and EUGENIO BULYGIN, *Normative Systems*, Springer-  
 689 Verlag, Wien, 1971.
- 690 [2] ANDERSON, ALAN ROSS, ‘A reduction of deontic logic to alethic modal logic’, *Mind*,  
 691 67 (1958), 100–103.
- 692 [3] ASHER, NICHOLAS, and DANIEL BONEVAC, ‘Free choice permission is strong permis-  
 693 sion’, *Synthese*, 145 (2005), 3, 303–323.
- 694 [4] BACH, KENT, ‘Actions are not events’, *Mind*, 89 (1980), 353, 114–120. New series.
- 695 [5] BARKER, CHRIS, ‘Free choice permission as resource-sensitive reasoning’, *Semantics*  
 696 *and Pragmatics*, 3 (2010), 10, 1–38.
- 697 [6] BEARDSLEY, ELIZABETH LANE, ‘Imperative sentences in relation to indicatives’,  
 698 *Philosophical Review*, 53 (1944), 2, 175–185.
- 699 [7] CASTAÑEDA, HECTOR NERI, *Thinking and Doing*, D. Reidel Publishing Co., Dordrecht  
 700 and Boston, 1975.
- 701 [8] CHARLOW, NATE, ‘Restricting and embedding imperatives’, in Maria Aloni, and  
 702 Katrin Schulz, (eds.), *Proceedings of the 17th Amsterdam Colloquium*, vol. 6042 of  
 703 *LNAI*, Springer-Verlag, Berlin, Heidelberg, 2010, pp. 223–233.
- 704 [9] CLARK, BILLY, ‘Relevance and “pseudo-imperatives”’, *Linguistics and Philosophy*, 16  
 705 (1993), 79–121.
- 706 [10] DIGNUM, FRANK, JOHN-JULES CH. MEYER, and ROEL WIERINGA, ‘Free choice and  
 707 contextually permitted actions’, *Studia Logica*, 57 (1996), 1, 193–220.
- 708 [11] FOX, CHRIS, ‘Obligations, permissions and transgressions: an alternative approach to  
 709 deontic reasoning’, in *Proceedings of the Tenth Symposium on Logic and Language*,

- 710 Theoretical Linguistics Program, ELTE, Budapest, Balatonszemes, Hungary, 2009,  
711 pp. 81–88.
- 712 [12] FRANKE, MICHAEL, ‘How and how not to employ discourse relations to account for  
713 pseudo-imperatives’, in Paul Dekker, and Michael Franke, (eds.), *Proceedings of the*  
714 *Amsterdam Colloquium*, ILLC/Department of Philosophy, University of Amsterdam,  
715 Amsterdam, 2005, pp. 83–88.
- 716 [13] FRANKE, MICHAEL, *Pseudo-Imperatives*, Master’s thesis, Institute for Logic, Language  
717 and Computation, University of Amsterdam, 2005.
- 718 [14] GINZBURG, JONATHAN, and IVAN SAG, *Interrogative Investigations*, CSLI, Stanford,  
719 2000.
- 720 [15] HAGE, JAAP C., *Studies in Legal Logic*, chap. 6: What is a Norm?, Springer, Berlin,  
721 2005, p. 173ff.
- 722 [16] HAMBLIN, CHARLES L., *Imperatives*, Blackwell, Oxford, 1987.
- 723 [17] HAN, CHUNG-HYE, *The structure and interpretation of imperatives: mood and force*  
724 *in universal grammar*, Outstanding Dissertations in Linguistics, Garland, New York,  
725 2000.
- 726 [18] HAN, CHUNG-HYE, ‘Imperatives’, in Claudia Maienborn, Klaus von Heusinger, and  
727 Paul Portner, (eds.), *Semantics: An international handbook of natural language*  
728 *meaning*, Mouton de Gruyter, to appear.
- 729 [19] HARE, RICHARD MERVYN, ‘Imperative sentences’, *Mind*, LVIII (1949), 21–39. Also in  
730 [21].
- 731 [20] HARE, RICHARD MERVYN, ‘Some alleged differences between imperatives and indica-  
732 tives’, *Mind*, LXXVI (1967), 303, 309–326.
- 733 [21] HARE, RICHARD MERVYN, *Practical Inferences*, Macmillan, 1971.
- 734 [22] HUNTLEY, MARTIN, ‘The semantics of English imperatives’, *Journal of Linguistics*  
735 *and Philosophy*, 7 (1984), 103–133.
- 736 [23] JACKSON, FRANK, ‘On the semantics and logic of obligation’, *Mind*, 94 (1985),  
737 177–195.
- 738 [24] JØRGENSEN, JØRGEN, ‘Imperatives and logic’, *Erkenntnis*, 7 (1937–38), 288–296.
- 739 [25] KAMP, HANS, ‘Free choice permission’, *Proceedings of the Aristotelian Society*, 74  
740 (1973), 57–74.
- 741 [26] KAMP, HANS, ‘Semantics versus pragmatics’, in Franz Guenther, and Siegfried J.  
742 Schmidt, (eds.), *Formal Semantics and Pragmatics for Natural Language*, Synthese  
743 Language Library, D. Reidel, 1979, pp. 255–287.
- 744 [27] KLEIN, DANIEL B., and BRENDAN O’FLAHERTY, ‘A game-theoretic rendering of  
745 promises and threats’, *Journal of Economic Behavior and Organization*, 21 (1993),  
746 295–314.
- 747 [28] KRIFKA, MANFRED, ‘Semantics below and above speech acts’, Talk held at Stanford  
748 University, 2004.
- 749 [29] LAPPIN, SHALOM, ‘On the pragmatics of mood’, *Linguistics and Philosophy*, 4 (1982),  
750 559–578.
- 751 [30] LASCARIDES, ALEX, and NICHOLAS ASHER, ‘Imperatives in dialogue’, in Peter Kühn-  
752 lein, Hans Rieser, and Henk Zeevat, (eds.), *The Semantics and Pragmatics of Dialogue*  
753 *for the New Millennium*, Benjamins, 2004, pp. 1–24.
- 754 [31] LEWIS, DAVID, ‘General semantics’, in Donald Davidson, and Gilbert Harman, (eds.),

- 755           *Semantics of Natural Language*, Reidel-Dordrecht, 1972, pp. 169–218.
- 756 [32] MASTOP, ROSJA, *What can you do? — Imperative Mood in Semantic Theory*, Ph.D.  
757 thesis, University of Amsterdam, 2005.
- 758 [33] MCCARTHY, JOHN, and PATRICK J. HAYES, ‘Some philosophical problems from the  
759 standpoint of artificial intelligence’, in Bernard Meltzer, and Donald Michie, (eds.),  
760 *Machine Intelligence*, vol. 4, Edinburgh University Press, 1969, pp. 463–502.
- 761 [34] PIWEK, PAUL, ‘Imperatives, commitment and action: Towards a constraint-based  
762 model’, *LDV Forum: GLDV-Journal for Computational Linguistics and Language  
763 Technology, Special Issue on Communicating Agents*, 17 (2000), 1,2. ISSN 0175–1336.
- 764 [35] PORTNER, PAUL, ‘The semantics of imperatives within a theory of clause types’,  
765 in Kazuha Watanabe, and Robert B. Young, (eds.), *Proceedings of Semantics and  
766 Linguistic Theory 14*, CLC Publications, Ithaca, NY, 2005. Paper presented at SALT  
767 14, 14th–16th March 2004.
- 768 [36] PORTNER, PAUL, ‘Permission and choice’, manuscript available online from  
769 [semanticsarchive.net](http://semanticsarchive.net), 2010.
- 770 [37] PÉREZ-RAMÍREZ, MIGUEL, and CHRIS FOX, ‘An axiomatisation of imperatives using  
771 Hoare logic’, in Harry Bunt, Ielka van der Sluis, and Roser Morante, (eds.), *Proceedings  
772 of the Fifth International Workshop on Computational Semantics (IWCS-5)*, Tilburg,  
773 Netherlands, 2003, pp. 303–320.
- 774 [38] ROSS, ALF, ‘Imperatives and logic’, *Theoria*, 7 (1941), 53–71. Republished as [39].
- 775 [39] ROSS, ALF, ‘Imperatives and logic’, *Philosophy of Science*, 11 (1945), 30–46.
- 776 [40] RUSSELL, BENJAMIN, ‘Imperatives in conditional conjunction’, *Natural Language  
777 Semantics*, 15 (2007), 2, 131–166.
- 778 [41] SCHMERLING, SUSAN F., ‘How imperatives are special and how they aren’t’, in Robin-  
779 son Schneider, Kevin Tuite, and Robert Chametzky, (eds.), *Papers from the Chicago  
780 Linguistics Society (CLS) Para-Session on Nondeclaratives*, Chicago Linguistics Soci-  
781 ety, University of Chicago, 1982, p. 93–106.
- 782 [42] SCHWAGER, MAGDELENA, *Interpreting imperatives*, Ph.D. thesis, University of  
783 Frankfurt-Main, 2006. Under revision for the Springer series “Studies in Linguis-  
784 tics and Philosophy”.
- 785 [43] SCONTRAS, GREGORY, and EDWARD GIBSON, ‘A quantitative investigation of the  
786 imperative-and-declarative construction in english’, , 2010. Manuscript, Harvard/MIT.
- 787 [44] SEGERBERG, KRISTER, ‘Validity and satisfaction in imperative’, *Notre Dame Journal  
788 of Formal Logic*, 31 (1990), 2, 203–211.
- 789 [45] STEFAN KAUFMANN, MAGDALENA SCHWAGER, ‘A unified analysis of conditional  
790 imperatives’, in Ed Cormany, Satoshi Ito, and David Lutz, (eds.), *Proceedings of  
791 the Semantics and Linguistic Theory Conference (SALT) 19*, eLanguage, 2011, pp.  
792 239–265. Conference held 3rd–5th April 2009 at The Ohio State University.
- 793 [46] STENIUS, ERIK, ‘Mood and the language game’, *Synthese*, 17 (1967), 254–274.
- 794 [47] TURNER, RAYMOND, *Computable Models*, Springer, 2009.
- 795 [48] VAN EIJCK, JAN, ‘Making things happen’, *Studia Logica*, 66 (2000), 1, 41–58.
- 796 [49] VON FINTEL, KAI, and SABINE IATRIDOU, ‘Morphology, syntax, and semantics of  
797 modals’, Lecture notes for the Linguistics Society of America (LSA) summer institute  
798 at Berkeley, 2009.
- 799 [50] WYNER, ADAM ZACHARY, *Violations and Fulfillments in the Formal Representation*

- 800            *of Contracts*, Ph.D. thesis, King's College London, 2008.  
801            [51] ZANUTTINI, RAFFAELLA, 'Encoding the addressee in the syntax: evidence from english  
802            imperative subjects', *Natural Language and Linguistic Theory*, 26 (2008), 1, 185–218.  
803            [52] ZIMMERMANN, THOMAS EDE, 'Free choice disjunction and epistemic possibility',  
804            *Natural Language Semantics*, 8 (2000), 255–290.

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