Mass Terms and Plurals in Property Theory*

Chris Fox†
University of Essex
Wivenhoe Park
Colchester
U.K.
Email: foxcj@uk.ac.essex

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Abstract

This paper is concerned with representing the semantics of natural language (NL) plurals and mass terms in Property Theory (PT): a weak first-order theory of Truth, Propositions and Properties with fine-grained intensionality Turner, 1990; Turner, 1992; Aczel, 1980.

The theory allows apparently coreferring items to corefer without inconsistency. This is achieved by using *property modifiers* which keep track of the property used to refer to a term, much like Landman's *roles* Landman, 1989. We can thus predicate apparently contradictory properties of "the judge" and "the cleaner", for example, even if they turn out to be the same individual.

The same device can also be used to control distribution into mereological terms: when we say "the dirty water is liquid", we can infer that those parts that are dirty water are liquid without inferring that the dirt is liquid.

The theory shows how we can formalise some aspects of natural language semantics without being forced to make certain ontological commitments. This is achieved in part by adopting an axiomatic methodology. Axioms are proposed that are just strong enough to support intuitively acceptable inferences, whilst being weak enough for some ontological choices to be avoided (such as whether or not the extensions of mass terms should be homogeneous or atomic). The axioms are deliberately incomplete, just as in basic PT, where incomplete axioms are used to avoid the logical paradoxes.

The axioms presented are deliberately too weak to say much about 'non-denoting' definite descriptors. For example, we cannot erroneously prove that they are all equal. Neither can we prove that predication of such definites results in a proposition. This means that we cannot question the truth of sentences such as "the present king of France is bald".

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1 Introduction

In essence, this paper presents a semantic theory which:

(i) Allows apparently coreferring terms to corefer whilst permitting different, possibly contradictory predicates to apply to those terms without resulting in inconsistency. For example, "the cleaner" and "the judge" may have different incomes, even though both positions may be taken by one person;

(ii) Allows the distribution of properties into *mereological* (part-whole) terms without distributing to inappropriate parts. We might say "the dirty water is liquid", and infer that parts of the dirty water are liquid, without inferring that the dirt is liquid;

(iii) Gives a treatment of so-called 'non-denoting' definites within a classical, two-valued theory which does not require that non-denoting terms be equated, so that "the present king of France", and "the largest prime number" can be distinguished in the semantics.

Points (i) and (ii) are intimately related. They stem from an attempt to devise a semantic theory of plurals and mass terms with a minimum of ontological commitments.

It is always possible both to block inappropriate distributive inferences, and to allow different properties to hold of apparently coreferring terms by adopting a highly intensional ontology. In such an ontology, dirt is not (in a formal sense) a part of dirty water, and a judge (who is John) is represented by some term distinct from the individual denoted by "John". However, part of the motivation behind this paper is to see just how far we can go in preventing inappropriate inferences of these kinds without presupposing such an ontology.

This is not to say that in devising a semantic theory we should never make ontological commitments. However, we should be cautious in asserting that certain commitments are *necessary* to explain the empirical data, when they might only be required by a particular analysis, perhaps to make up for some undesirable artifact of the chosen formalisation.

It is in this sense that point (iii) is related to (i) and (ii). It is concerned with constraining inferences without requiring a special element in the domain for 'non-denoting' definite descriptors, or a three-valued theory.

All three themes are concerned with intensionality of subject noun phrases. They involve the issue of distinguishing uses of noun phrases that appear to corefer on an unsophisticated extensional account.

1.1 Intensional Subjects

The intensionality of subject position noun phrases is apparent with sentences involving collective nouns such as "committee". For example, we might have two different committees with the same members. We can predicate different properties of these two committees without fear of contradiction; if one of the committees is having a meeting it does not mean that the other committee with the same members is also having a meeting Bennett, 1977; Link, 1984.

As Landman points out, similar issues arise with singular nouns. For example, if we have a judge and the cleaner, we appear to be able to say:

The judge earns £20000.
The cleaner earns £2000.

without contradiction, even if one individual, John, is both judge and cleaner Landman, 1989. Landman argues that the definite noun phrases cannot be taken to simply denote the relevant individual which then has the property denoted by the verb phrase applied to it.

These arguments parallel those exemplified by the Evening Star-Morning Star problem. The Morning Star and the Evening Star both refer to the planet Venus. On an unsophisticated account these definite descriptions might then be equivalent. However, we can say:

> The Morning Star and the Evening Star were once thought to be different.

without meaning that the Morning Star was once thought to be different from itself. This sentence differs from the judge-cleaner example in that it involves a propositional attitude.

Landman suggests that the individuals referred to in these sentences are really individuals under certain roles. An individual may have different properties under different roles. The properties an individual has under one role may contradict those it has under another role. This may be made explicit in English by such sentences as:

> John, as a judge, earns £20000.
> John, as a cleaner, earns £2000.

He proposes a new, distinct ontology for the denotations of individuals acting under different *guises* or *roles*. Landman effectively predicates "earns £20000" of *John-as-a-judge*, and "earns £2000" of *John-as-a-cleaner*, where *i-as-a-p* is a new kind of intensional individual.

Landman adopts a strongly typed theory where the individuals acting under guises cannot be taken to be new individuals themselves.[1] As a consequence, when dealing with plurals, Landman requires not only a lattice-theoretic structure over the domain of individuals, following Link for example Link, 1983; Link, 1984; Link, 1987; Link, 1991b, but also a similar, but separate structure over the intensional individuals.

Landman suggests that it might be fruitful to consider a theory where his 'individuals under guises' are considered to be individuals themselves[2], and also to use a more weakly typed theory such as Ray Turner's axiomatisation Turner, 1987; Turner, 1990; Turner, 1992 of Aczel's Frege Structures Aczel, 1980. These options, together with the property modifier treatment, are explored in more detail elsewhere Fox, 1993; Fox, 1994c.

Given the sentence:

> The judge is strict.

where John is the judge, it might seem more natural to say "John is a strict judge" than "John, as a judge, is strict". The sentence "the judge is strict" could well be taken to be elliptic for "the judge is a strict judge". Even when the "as a ..." construction is natural, the use of commas, when it is adjacent to the nominal, suggests some *movement* has occurred:

> John, as a judge, earns £20000.
> John earns £20000 as a judge.

This, perhaps, gives some support to the policy of treating roles as property modification (or, in this presentation, as properties modified by the verb phrase) rather

[1] If individuals are of type e and properties are of type p, then Landman's new individuals-under-guises are taken to be of the same type as quantifiers, $\langle\langle e, p\rangle, p\rangle$, rather than of type e.

[2] That is, individuals-under-guises are taken to be of type e.

than modification of the individual, although syntactic arguments like this are potentially hazardous.

Following this, the judge-cleaner example can be treated by assuming the predication is modified, rather than the individual. So we would have the properties "(earns £20000)-as-a-judge" and "(earns £2000)-as-a-cleaner". The property modifiers will correspond, in some sense, to the *subsective adjectives* Kamp, 1975 even though in the final theory they may be derived from the translation of natural language verbs.[3] The term they modify shall usually be derived from the subject nominal, so we would have, in effect:

> John is a £20000 earning judge.
> John is a £2000 earning cleaner.

This approach means that only one plural domain is required as there are no new 'individuals under guises'.

Landman argues in favour of 'intensional individuals', over some form of modification to the predication, on the basis of examples involving comparatives, such as:

> The judge and the cleaner have different incomes.

where, again, John might be both judge and cleaner.

Presumably he sees this as a problematic example for a property modifier approach as there only appears to be one property—"have different incomes"—yet there would be two property modifiers. That is, on a property modifier approach, the interpretation of the verb would have to provide for as many modifiers as there are constituent noun phrases in the subject. If roles modify the interpretation of the noun phrases, then, trivially, there are always the same number of objects to be modified as there are modifiers. However, the appearance of only one verb phrase in the natural language sentence does not preclude there being more than one appropriate property in the semantic representation of the sentence. Landman's objection must rest on the assumption that "have different incomes" is represented by some single, irreducible property in the semantics. In particular, his objection must assume that there can be no useful reduction of the phrase modified by the adverb "different". If, in the truth conditions of the sentence, we can provide a property to be modified for each noun phrase, then an interpretation can be given to the sentence using property modifiers. We could paraphrase the comparative sentence as:

> The income *from cleaning* earned by the person who is the cleaner is different to the income *from judging* earned by the person who is the judge.

with the outcome that:

> John's income as a judge is different to his income as a cleaner.

if John is both judge and cleaner.

So, it does seem that Landman's problematic example *can* be treated by way of a more complex predication, provided care is taken with the analysis of the comparative itself Fox, 1994c; Fox, 1993, as indicated in Section 4. As a consequence, the examples involving the intensionality of subject position noun phrases can be treated without complicating the ontology. Apparently coreferring items can be allowed to corefer whilst possessing different properties.

[3]Subsective adjectives have also been called *restrictive* or *affirmative* adjectives Hoepelman, 1983.

This can also be used to treat examples involving reference to an object and the material of which it is made. Thus, given a ring made from gold it is possible to say:

The gold is old.
The ring is new.

without contradiction, as "the gold" in effect refers to an object under the guise of being gold, and "the ring" may refer to the same object under the guise of being a ring.[4]

In the final theory, property modifiers are used to control inferences exemplified by distribution, whilst allowing (though not requiring) an inhomogeneous ontology; dirt and water may be parts of dirty water, for example. Inferences involving distributive properties can cause problems with such ontologies, because we want to distribute to the correct parts. Landman's insights enable us to embellish the application of a property to a term so that we can recover these parts. This is elaborated in Section 1.2.

In general, how do we know under which aspect the application of a property to a term should be restricted? That is, in Landman's terminology, where do *roles* come from? With sentences where the subject is not a proper name, one possibility suggests itself: if we take "the man is tall", then the subject noun phrase can provide the information that enables us to interpret this as "the man is a tall man". However, a sentence may be ambiguous as to whether such a device is appropriate:

The cleaners are badly paid.

may mean either that the cleaners are badly paid cleaners, or that the cleaners are badly paid people. If we take the latter reading, where does the aspect of being a person come from? Perhaps a theory of context and salience could provide the answer. I see several possibilities:

(i) take "badly paid" as ambiguous between being *badly paid as an X* where the subject fills in the X and just plain *badly paid (as a person)*;
(ii) take "The cleaners" as ambiguous between *the cleaners as cleaners*, and *the cleaners as individuals*;
(iii) have some sortal hierarchy, which says *a cleaner is a person*, and allow individuating terms, optionally, to be generalised;
(iv) some other, more general, theory of context and salience.

It should be noted that, under whichever role we treat the cleaners, we can obtain the correct individuation. It is not, however, possible to take the sentence as ambiguous between providing a role and not, since we must know how to obtain the appropriate nice parts.

Considering sentences involving proper names, when we say that "John is tall" and "Everest is tall", it should be clear that we are not asserting that John and

[4] It might seem that the gold and the ring can be distinguished because "the ring" individuates, and "the gold" does not. However, in general it is not clear that there is always a means to distinguish such terms, as with the Morning Star-Evening Star. Also, see the later argument on wood-furniture and food-potatoes in Section 1.2.

Landman also applies this treatment of intensionality to the 'committee' example. For him, the two committees are represented by the same object (the collection of members) under different guises. Although there should be some systematic connection between the committees and their members, it seems counter-intuitive to make the collection of members a constituent part of the term representing the committees: if the members change then the committees are no longer the same, and we have an identification problem. Barker's treatment of collectives seems more appropriate Barker, 1992. He has some function that returns the members of a committee, but the terms representing committees themselves are independent of their members. Here is one case where linguistic examples require a particular ontology.

Everest are both in the class of tall objects. Presumably, we mean "John is a tall person" and "Everest is a tall mountain". These could then be taken as forms of ellipsis. But what information do we use to complete the interpretation? The translation of proper nouns can include the role under which they are to be considered, so we could have, in some sense, *the person John* and *the mountain Everest* as our representations of these names. These considerations are the same as those that arise in the interpretation of adjectives Kamp, 1975; Montague, 1974. The final theory (Section 4) is only concerned with sentences involving quantified noun phrases. Their semantic representation gives rise to some appropriately modified predication in the truth conditions.[5]

If guises are added to the theory, either with the aid of property modifiers, or new terms, then presumably these can be inherited by anaphoric reference. For example:

> There is [a judge]$_i$.
> [He]$_i$ is strict.

should gives rise to:

> There is a strict judge.

This would seem easier to formalise if Landman's restricted individuals are used, rather than property modifiers, as this would be a simple extension of existing discourse theories (for example Kamp, 1981; Heim, 1982; Kamp and Reyle, 1993). However, the property modifiers need only arise in the truth conditions. If the resolution of the anaphora occurs before this stage, then there may be no call for additional complexity.

1.2 Mass Terms and Distribution

Sentences containing mass terms provide another class of examples where it seems that we might wish to allow nominals to corefer, whilst requiring that different properties hold of them. This becomes apparent with distributive inferences. If we apply a property to a term, the property is said to be distributive if it holds of the parts of that term. With count nouns we know that we should only distribute at most as far as the individuals. With mass terms, consideration of natural language alone does not always make it obvious when to stop performing distributive inferences. If we have some water, and we say it is liquid, then we would like to infer that parts of it are also liquid. Clearly, we only want those parts of it that are water to be considered liquid, rather than individual molecules or fragments of such. The problem is that in the conventional compositional analysis, we cannot gain access to the property that indicates the appropriate parts ("water" in this case) unless we put constraints on the denotation of "water".

The problem can be demonstrated without appeal to the atoms of physics. If we have some dirty water, we can say that "the dirty water is liquid" and expect to be able to infer that parts of the dirty water are liquid, without inferring that some dirt is liquid. Here, however, limits on the appropriate inferences are discernible from our intuitions about language. We are interested only in distributing properties to

[5] The sentence "a cleaner strikes" could be represented as something like:

$$\exists x(\text{cleaner}'x \wedge (\text{strikes}'(\text{cleaner}'))x)$$

This is true should there be a cleaner who is a striking cleaner (or alternatively, who is striking, as a cleaner).

the 'nice' parts of terms, where the nice parts of some cats, for example, are cats, rather than bits of cats Lewis, 1991.[6]

We can avoid distributing properties to the wrong parts if we constrain the ontology so that parts of water are water and parts of dirty water are dirty water. This is an homogeneous ontology, which is intended to be argued for, in part, by Bunt's *homogeneous reference hypothesis*:

> "A mass noun refers in such a way that no particular articulation of the referent into parts is presupposed, nor the existence of minimal parts."
> Bunt, 1985

If we were pedantic, we might argue that an homogeneous ontology *does* presuppose a particular articulation of the referents into parts. We shall look at the question of minimal parts below in Section 1.3.

Although an homogeneous ontology avoids distributing to inappropriate parts (as there are no inappropriate parts) it should not be taken as a conclusive argument in favour of such an ontology. The reason we might consider inappropriate parts in an inhomogeneous ontology is that in the representation, the property used to denote the term in question is present only by way of its extension (or the supremum of its extension). If the suprema, or fusions, of the extensions of two distinct properties are equal, then distributing into one will be equivalent to distributing into the other, even though the two properties may hold of different parts. If we allow the suprema to be equal, and we wish to control distributive inferences, then we need some way of keeping track of the property whose suprema we are distributing into.

There is a sense in which theories that give homogeneous extensions to mass terms have effectively encoded the property used to denote a mass term in the extension. However, Landman's 'individuals under guises' provide an alternative, explicit way of associating the property that has been used to denote with the denotation. This property can then be used to control distributive inferences with mass terms. If we implement Landman's intensional individuals using property modifiers, then we can control distributive inferences with mass terms using an ontology of plain individuals, and some Boolean algebra-like structure that need not be homogeneous. As an example, we might interpret the sentence "all water is liquid", as predicating "liquid-water" of the mereological sum of all water. That predicate can then distribute to those parts that are water (as opposed to arbitrary parts of the sum).

There is a potential problem with this, depending upon our view of the meaning of "liquid". For this treatment of distribution to work, it must be assumed that the smallest denotables that have the property of being "water" must still be large enough to possess any distributive properties. This treatment of distribution would fail if we consider a molecule of water to be water, yet only allow a more substantial body of water to be liquid. Further, if we consider "being liquid" to be a property of a collection of molecules that are a physically continuous body, then this account would fail if denotable water could consist of the fusion of discontinuous fragments of water, each fragment being too small to be attributed the property of being liquid.[7] Such criticisms seem to assume a reductive account of the extension of mass terms to notions in modern physics. We can maintain that the denotables that we can refer to by a term like "water" are precisely those terms to which

[6]This observation was also made by Quine. He says that we only want to distribute to those parts that are not "too small" to count as the appropriate sort of term Quine, 1960. Roeper makes a related point concerning the distribution of disjunctive properties Roeper, 1983, as discussed below. Lønning also addresses the issue of inappropriate parts as they arise in sentences involving negation Lønning, 1987.

[7]These criticisms were suggested to me by Hans Kamp

we can attribute properties such as "being liquid". We could view an attempt to ascertain the smallest such denotable in terms of physical molecules as rather like trying to give a reductive account of the meaning of "a table" in terms of a physical arrangement of components.

We might look more closely at the notion of smallest part in modern physics. It tells us that the smallest element of water is a water molecule, precisely because a water molecule is the smallest physical component of water that possesses the properties that distinguish it as water for the purposes of theories in physical science. If we accept that the property of "being liquid" cannot be attributed to molecules in isolation, then for the purposes of this theory of semantics, the smallest component of water that counts as water will have to be something more than a molecule. This does not prevent reference to *molecules of water*, as this sentence demonstrates. According to this theory, we might then say "the water is liquid" without entailing that "the molecules of water are liquid".

Notice that we can use the same machinery to control distribution with plurals.

In order to stop distribution at an appropriate point, it is not necessary to use the full intensionality of the property modifier regime: all that is required is for the property which is modified to provide the correct individuation. Any additional intensionality can, in principle, be thrown away. Thus we can always consider cleaners as cleaners, and John as John. However, Landman provides sufficient indication as to why we might not want to do this.[8]

Even if we can constrain the distribution of simple properties to appropriate 'nice' parts, there are added complications with disjunctive properties, as noted by Roeper Roeper, 1983. For example, with the sentence:

> All phosphorus is red or white.

it is not sufficient to be able to pick out the nice parts of phosphorus and say of them that they are red, or that they are white, because there may be some bits of phosphorus that are both red and white. There are also similar complications with negation Lønning, 1987. Clearly, care must be taken when giving the truth conditions of such examples. These problems would also affect a treatment with homogeneous extensions.

It could be argued that if we were to adopt an homogeneous ontology then there would be no identification of, for example, "the gold" and "the ring" in "the gold ring", as they have different parts. Indeed the ring has no proper parts that constitute a complete ring. This is an argument originally intended to be against the identification of substances (the denotation of mass terms) and physical matter, attributed to Quine by Parsons:

> "even if all and only furniture was composed of wood, it would not follow that wood = furniture, since parts of the chairs might be wood without being furniture." Parsons, 1970.

It can be taken as an argument against the need to keep track of the property used to refer, and in favour of a particular ontology for the denotations of noun phrases.

Bunt disputes the wood-furniture example, suggesting that the problem lies in taking "furniture" to be a mass term with minimal parts. He cites the same example, replacing the individuating term "furniture" with "sawdust", so that all the wood in the world is made into sawdust. He then says that "wood" and "sawdust" can legitimately be equated, in non-intensional contexts Bunt, 1985. However, this

[8] It may be noted that theories of individuals under different roles may allow descriptors with no extension in the denotable terms to denote the bottom element $\perp$. The bottom element could then be considered under different guises, preventing different 'non-denoting' terms being equated.

cannot be the case, as we might truthfully say "this *sawdust* was made yesterday", without commitment to "this *wood* was made yesterday".[9]

It might still be argued that the sawdust and wood are distinguishable because they have different nice parts: there are parts of the sawdust that are not sawdust, but are wood. However, it is conceivable that some sawdust might be sufficiently finely ground for its minimal nice parts to constitute the minimal nice parts of wood, but we would still want to be able to predicate different things of the sawdust and wood. So, even if an ontology is adopted where terms are equated only if they have the same minimal parts, there is still some motivation for using Landman's intensional individuals, or property modifiers.

As stated before, in the final analysis it might turn out that we really do need an highly intensional ontology where apparently coreferring terms do not corefer. However, the notion of a term being considered under a guise does permit such coreference. Again, if terms under guises are represented using modified predications, then constraints on appropriate distributive inferences can be guaranteed without additional ontological commitments other than some part-whole structure in the domain.

There have been previous attempts to provide a formalisation of part-whole structures that do not assume an homogeneous ontology. Moravcsik gives two theories that seek to do just this Moravcsik, 1973.

He represents predication with the part-of relation. In the first theory, "x is water" is represented by:

$$x \leq \text{water}'_{sp}$$

where water'_{sp} consists of the objects not too small to count as water. This explicitly creates an homogeneous term from a potentially inhomogeneous one. Unfortunately, representing predication in this way means that all mass terms must be homogeneous Fox, 1993 on pain of inconsistency Bunt, 1985, as the representation of the sentence "water is water" ($\text{water}' \leq \text{water}'_{sp}$, where water' is the potentially inhomogeneous mereological sum of "water") requires that all unconstrained parts of water are not too small to count as water.

In his second theory, Moravcsik seeks to constrain the part-of relation to make it homogeneous in character. The expression:

$$x \leq_{sp(m)} m$$

is used to indicate that x must be a part of m not too small to count as m. Or, to express it more generally, it must be either a *nice* part of m, or a sum of *nice* parts. As it stands, this account does not deal with all the data. If we have:

The puddle is water.
Water is wet

then we should be able to prove that the puddle is wet. If predication is modelled by the part-of relation, then in this case we require transitivity across the different kinds

[9]If this is ruled inadmissible in an extensional theory, as it is some 'intensional context', then we might question what a '*non*-intensional context' could be.

In whatever way we arrange the example, wood and sawdust definitely do seem to be different. If property modifiers, or Landman's roles, are not used, then an intensional ontology may be required. However, the theory presented in this paper is weak enough to allow the extensions of these expressions to be equated.

Two definite descriptions (brought to my attention by an anonymous referee) where there might not be such an obvious distinction as there is between "wood" and "sawdust", are "the food" and "the potatoes", where the potatoes (read as a mass term) exclusively constitutes the food. If there is necessarily no example which distinguishes between these expressions, then meaning postulates would have to be added to that effect.

of part-of relations Bunt, 1985. Moravcsik does not elaborate on the relationships between the different orderings.

The arguments in this paper are concerned only with the ontology of things that are the denotations of natural language, rather than with some reductive ontology of the physical world. For example, I will take it that a cat can be the same cat even if it looses a whisker, following the familiar arguments of Aristotle Burge, 1975, and that part-whole structures in the denotation of natural language can survive physical mereological change Cartwright, 1965, thus some water can be the same water even after some of it evaporates. Also, if there is some dirty water, it may remain the same dirty water even if the dirt is in dynamic equilibrium with some sludge at the bottom of the water.

This paper does not attempt to address the general problem of identification of substances and objects. There may be good arguments for not identifying a gold ring with the gold that it is made from. On the other hand, there are examples where different descriptors surely do denote the same object, such as "the Evening Star" and "the Morning Star".

The final theory presented in this paper will allow extensions to be equated, and terms to be part of other terms, without equating definite descriptors or allowing unmotivated distributive inferences. By using property modifiers, some aspects of a highly intensional homogeneous ontology can be obtained in a more extensional one.

1.3 Atomicity

For mass terms, an additional issue is whether the part-whole structure in the domain should be *atomic*. If we are only dealing with count terms, then there seem to be good arguments for an atomic domain, in particular, we can refer to minimal parts with singular count nouns.

With mass terms, the answer it is not so clear. Although modern physics tells us that everyday substances are atomic, this need not be reflected in the ontology required for natural language semantics. Indeed, as non-linguistic research was required to demonstrate the atomicity of physical substance, it would be strange then to argue that atomicity can be derived solely from the semantics of natural language. Further, there are abstract things that we refer to by way of non-count terms which might not be atomic, such as space and time.

It seems that natural language semantics is consistent regardless of whether the denotations of mass terms are taken to be atomic or not. This suggests that a formal theory of mass terms should be incomplete in this respect: we should neither be able to prove that mass terms are atomic nor that they are atomless.

The theory presented in this paper strives to show that we can obtain a consistent theory of mass terms, and plurals, which works whether or not the domain is taken to be homogeneous or atomic. There may be issues that force us to decide between these ontologies, but we seem to be able to go quite a long way without making such choices.

1.4 Axiomatic Semantics

It might be argued that some position has to be adopted on the point of atomicity, for example, because the rigours of model-theoretic semantics demand it. If we view the formal semantics programme as consisting of natural language data and its model, then this might be correct.

When examining new phenomena in natural language, the model-theoretic approach is to look for some mathematical structure, or system, which appears to display the appropriate behaviour. This structure is then used as a model for the formal semantics of natural language. The representation language is made to adopt the behaviour of this model by asserting that all the entailments of the model are legitimate inferences in the representation (completeness), and all inferences in the representation are supported by the model (soundness).

By insisting that the entailments of the representation are precisely those of the model, there is a danger that artifacts of the model may lead to too strong a logic. For example, if the model employs a type hierarchy to avoid self application paradoxes, as in Montague semantics, then the constraints of the strongly typed regime are inherited by the semantic representation, even if not justified by (or even counter to) considerations of natural language itself. Demanding completeness requires all issues to be decided one way or another, even when our intuitions provide no useful insight.

On this view, any intermediate representation language is considered merely as a formal aid in expressing generalisations Lønning, 1987. We can demonstrate that this intermediate language is redundant by showing there is a compositional mapping between it and the model, and that the language is sound and complete with respect to the model. Typically, a model for the semantics of mass terms will either be atomic or not. Completeness for the intermediate language will then mean that the representation must favour a particular ontology.

This suggests that, in model-theoretic semantics, we are forced to decide upon the ontology; we cannot leave such questions open in the semantic representation without making it incomplete with respect to a given model.

However, formally we need not demand that the intermediate representation language is complete with respect to any given model. The behaviour of the intermediate language—the inferences which it licenses—can be governed by axioms expressed in that language, rather than by adopting the inferences permitted in the model via completeness. We can give axioms that are too weak to make inferences about the ontology, such as whether it is atomic or homogeneous.

With this axiomatic methodology, we first establish the empirically accepted behaviour of natural language—in this case, natural language plurals and mass terms—and then axiomatise this in a suitable framework. Together with a suitable inference rule in the logic, such as *modus ponens*, these axioms allow proofs to be performed directly in the representation language. A model that satisfies these axioms can be found later. With this approach, the purpose of a model is merely to show that the theory is consistent.

Such axiomatic theories have already been proposed for the semantics of natural language. An example is the basic theory used in this paper: Turner's axiomatisation of Aczel's Frege Structures (PT). In PT, the notion of *proposition* is defined axiomatically (rather than syntactically). The axioms are too weak to prove that the logical paradoxes, such as the Liar, are represented by propositions. Only propositions may have their truth conditions considered. In this way, the theory allows self predication without being inconsistent. The theory is highly intensional: propositions are distinct from truth values (or sets of possible worlds); and properties are distinct from sets (or functions from individuals to sets of possible worlds).

The theory can be strengthened with axioms that satisfy our intuitions, as required. Only those aspects of behaviour of which we are certain need be axiomatised. Any term whose status is in doubt can be left unanalysed.

It is because of the fine-grained intensionality of PT that Landman suggests it might be a useful vehicle for formalising intensional individuals. PT seems a most suitable framework in which to express the semantic theory, as we require both some means of representing terms under guises, and also an axiomatic theory in order to avoid some of the commitments that might be required with model-theoretic semantics.

1.5 Definites

The motivation for the treatment of definite noun phrases given in this paper is related to issues of subject position intensionality. Theories of plurals and of mass terms often adopt something like a Boolean algebra to model part-whole relations. By definition, such algebras have a bottom element. As usually defined, this is the 'natural' denotation of 'non-denoting' definites. However, this has the undesirable consequence that all non-denoting terms are equated Parsons, 1970; Parsons, 1975. For example, on such an account "the present king of France" and "the largest prime number" would be the same even though their use indicates different existential presuppositions. Also, the theory must address the question of the nature of sentences containing such definites. In Russellian theories, predication of this bottom element is taken to result in a false proposition. Alternatively, a logic with either truth-value gaps or some 'unknown' truth value is used, in which case predication of the bottom element has either an indeterminate truth value, or some third unknown value Blau, 1981.

A *free logic* treatment would allow us to say that such descriptors denote elsewhere van Fraassen, 1966; Link, 1991a. The result of predication of such terms is undefined. Predication of non-denoting terms can be viewed as a variety of category mistake.

The property theory I shall use has been axiomatised to address much more awkward examples of category mistakes, namely the logical paradoxes that result from unconstrained self predication. As mentioned above, PT avoids the paradoxes by having axioms too weak to prove propositionhood of such sentences. Only those things which are propositions may have truth values (PT rejects bivalence). A similar position is open to us with non-denoting terms. We may have a class of denotable objects over which natural language quantifiers range. If a definite descriptor is represented as a supremum of denotables, then we can require (in the axioms) that the supremum is itself denotable only if the extension of the corresponding property is not empty. This achieves the effect of 'denoting elsewhere' in free logic. The undefinedness of predication of such terms can be obtained by requiring that all natural language derived properties are properties of denotables, and can only provably form propositions with terms that are provably denotable. We cannot prove that properties of denotables form propositions with non-denoting terms, and so we cannot consider their truth conditions. For example, the sentence "the present king of France is bald" will not provably be a proposition, and so it has no truth conditions.

We can also avoid equating non-denoting definite descriptors. Essentially, we can adopt the axioms of a Boolean algebra, but guard those axioms so that they only apply to 'denoting' terms. The axioms will then be too weak to prove that non-denoting terms are equal.[10] The representation of quantified noun phrases will contain quantifiers that are restricted to range over denotable terms.

Adopting the various views advocated models the effect of a free logic, without raising the question of what it means not to denote within a theory. Natural language predication of non-denoting definite descriptors is consigned to the same dustbin as other unhelpful constructions, such as the Liar sentence. It just becomes another case of a sortal category mistake.

In summary, axiomatic property theory avoids the paradoxes by having axioms that are too weak to prove that they are propositions. It is possible to adopt the same weak approach to both: (i) the nature of the denotation of so-called non-denoting definite descriptors; and (ii) the nature of the result of applying a property of denotables to such definite descriptors.

[10]The model for the theory may well contain a proper Boolean algebra, with a bottom element, but the theory itself will say nothing about it.

This is currently a static theory: non-denoting definite descriptors within a sentence will prevent a proof of propositionhood, no matter how deeply embedded.[11]

1.6 Other Issues

The theory developed here is not concerned with representing the various possible *intermediate* distributive readings, where properties are only distributed as far as certain sub-collections. In an intermediate distributive reading of the sentence:

> The boys and the girls met.

the verb "met" might be taken to distribute to the subcollections consisting of "the boys" and "the girls". Similarly with reciprocals:

> The boys and the girls hate each other.

we might wish to make explicit the relevant groups that hate each other. I only elaborate upon the fully distributive, and the fully collective readings. The collective readings are assumed to subsume any intermediate distributive readings.

One way in which these intermediate distributive readings have been tackled is by using *structured groups*, where the appropriate groups of a term are distinguished in some way, such as by non-associative brackets, or distinct kinds of summation, for example Hoeksema, 1983; Hoeksema, 1987; Higginbotham, 1981; Link, 1984; Roberts, 1987; Landman, 1989. However, I find Schwarzschild's arguments against structured-group representations to be convincing Schwarzschild, 1990; Schwarzschild, 1992. He argues, as does Gillon, that structuring the representation of the noun phrase using brackets, or non-associative conjunction does not give the correct interpretation in general, as some sentences require us to consider *minimal covers* rather than *partitions* of the extension of the noun phrase Gillon, 1992. Also, he shows that the correct structuring can depend upon properties used in the preceding discourse, and that obtaining this structure should not be thought of as the resolution of an ambiguity in the representations of noun phrases. This echoes some elements of the formal theory presented here, although the connection will not be explored.

In the theory presented, the operator that models conjunction is associative, and the operator that models the definite determiner also does not add structure to the representation of nominal terms. For simplicity, I do not adopt Schwarzschild's proposed context sensitive distribution operator that produces appropriate intermediate interpretations, nor do I offer a reductive interpretation of reciprocals.

The arguments for committee-like objects to be intensional, and distinct from their members is strong. To account for the different interpretation of sentences with these terms, it would be fairly straightforward to add ideas from Barker's theory Barker, 1992.

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[11] As has been indicated by Strawson, there are sentences involving non-denoting terms, to which some find it acceptable to ascribe a truth value. This is typified by passive constructions such as "The exhibition was visited by the present king of France" which some take to be a false proposition, unlike "The present king of France visited the exhibition" Strawson, 1964. One argument is that the existence presupposition only occurs for definite descriptors which are in the foreground, or active, in some sense. The theory presented does not preclude a strengthening of the axioms to allow propositionhood to be proved in such cases, should a suitable formal theory of the foreground/background distinction be forthcoming. A simple-minded response could be to give a Russellian, quantificational representation of passive definite descriptors, or to allow the passive form of "visit" to form propositions with terms that are not provably denotable. Lasersohn gives some other examples in which he claims the use of non-denoting terms is felicitous Lasersohn, 1993. A tentative treatment is offered for some of these examples is given elsewhere Fox, 1993.

The treatment of existential presuppositions with definites given in this paper can be extended to missing antecedents for anaphoric expressions Fox, 1994c.

The theory that I shall use for representing the semantics of natural language is essentially first-order. This means that considerations of how the scoping of accepted readings of sentences affects the logical power of English Lønning, 1989 cannot be considered within this theory. The problem of undecidable semantics arises when the semantic theory allows arbitrary fusions of (potentially infinite) collections which cannot be denoted by natural language nominals. In the theory to be adopted, only those collections that are in the extension of a denotable property may be considered. Thus, the issue of undecidability does not arise. This has consequences for the representation of mathematical statements: the statements of arithmetic cannot be expressed in this theory.

1.7 Outline of Paper

After presenting the property-theoretic framework (Section 2), I initially axiomatise a part-whole structure (Section 3) which does not make any assumptions about atomicity (minimal parts). It also characterises a notion of 'natural language denotable' that does not force non-denoting terms to be equated, nor force propositions involving non-denoting definites to be true or false, even though PT has a two-valued base logic. A model for this theory is given in the appendix.

This is the bare essentials for a part-whole structure in PT. It could be turned into a treatment of plurals by adding atomicity, or a treatment of mass terms by adopting an homogeneous ontology. I shall use this as a basis for the extensions of nominal expressions in the final theory (Section 4), which adds notions of singular and plural, and allows a unified treatment of mass terms and plurals.[12] The final theory also adds axioms for generalised conjunction.

A treatment of some of the puzzles in the literature is given, namely:

(i) simple distributive inferences with plurals;
(ii) the argument "all water is wet, the puddle is water, therefore the puddle is wet" Bunt, 1985;
(iii) the sentence "all water is water" Bunt, 1985;
(iv) the disjunction "all phosphorus is red or white" Roeper, 1983;
(v) the comparative "the cleaner and the judge earn different incomes" Landman, 1989 which, according to Landman, cannot be treated by a theory that uses property modifiers in place of individuals under guises.

This theory does not cover distributive inferences into conjoined noun phrases in order to avoid some combinatorial complexity in the presentation.

2 Propositions, Properties, and Truth

In this section, I present a theory of propositions, properties and truth (PT) due to Turner and Aczel Turner, 1990; Turner, 1992; Aczel, 1980. Before giving the general framework and the formal details of PT, I shall try and motivate why such a theory is useful for the semantics of natural language.

Many workers in this field use an extensional logic as a basis for their representations (Link, 1991a; Lønning, 1989 for example). In general, such extensional representations are inadequate for the semantics of natural language Chierchia, 1982; Chierchia, 1984; Chierchia and Turner, 1988. Intensionality is apparent in the case of opaque predication. The notion of possible worlds is often introduced into the representation language in order to account for this phenomenon. In such approaches, a proposition is a set of possible worlds, and a property is a function

[12] Unified in the sense that the extensions of mass terms and plurals can be part of the same part-whole structure, and axioms governing distributive inferences, for example, can apply to both.

from individuals to propositions. Possible worlds can be used to analyse modal notions such as possibility and necessity. Given an accessibility relation, *possibly p* is the set of worlds that have an accessible world in p. The possible worlds analysis has been extended to model knowledge and belief Hintikka, 1962; Kripke, 1963. This set-theoretic account fails on certain instances of propositional attitudes: an agent may know a certain proposition p, and there may be some proposition q, denoting the same set of possible worlds, yet we might not want to conclude that the agent knows q. The possible worlds account of knowledge and belief forces us to this unwanted conclusion. Such propositions are typified by those involving mathematical truths.

A further problem with possible worlds models is that they are typically strongly typed. Properties and relations can only hold of objects of a specific type. To generalise semantic notions across different types, type lifting, or type shifting has to be employed Partee and Rooth, 1983. The strong typing bars self predication and so avoids the paradoxes involving self-reference, such as the Liar: "this sentence is false". However, the strong typing also disallows unproblematic cases of self predication, such as "this sentence is six words long", and prevents the expression of universal properties.

These problems can be overcome if we treat propositions and properties as primitives. A property like "red" is not the set of red things, it is just itself, the property of being red. Similarly, the proposition, "$2 + 2 = 4$" is not merely a truth value, or a set of possible worlds, but it is a basic object, different from "$e^{i\pi} + 1 = 0$", even though, from the laws of mathematics, these propositions must always be true together.

PT exemplifies such a language: it has a highly intensional notion of properties and propositions which avoids the paradoxes without banning self predication through strong typing.

2.1 General Framework

Conceptually, PT can be split into two components, or levels. The first is a language of terms which consists of the untyped λ-calculus embellished with logical constants. A restricted class of these terms will correspond to *propositions*. When combined appropriately using the logical constants, other propositions result. As an example, given the propositions t, s, the conjunction of these, $t \wedge s$, is also a proposition, where $\wedge$ is a logical constant.

Some of the propositions will, further, be *true* propositions. When combining propositions with the logical constants, the truth of the resultant proposition will depend upon the truth of the constituent propositions. Considering the previous example, if t, s are both propositions that are true, then $t \wedge s$ will also be a true proposition.

There may be terms that form propositions when applied to another term. These terms are the properties. The act of predication is modelled by λ-application.

The essential point to note is that this is a highly intensional theory as the notion of equality is that of the λ-calculus: propositions are not to be equated just because they are always true together; similarly, properties are not to be equated just because they hold of the same terms (i.e., form true propositions with the same terms).

There are problems with the theory so far: the logical constants have no proof theory; and the notions of being a proposition, or a true proposition, cannot be expressed within this language of terms. That is, although we can consider terms as propositions, or true propositions, and comprehend how the propositionhood and truth of a term depends upon the propositionhood and truth of its constituent terms, we cannot express these notions formally *within* the language of terms: some *meta-language* is required. This is the purpose of the second component of PT: the language of *well formed formulae* (wff). This is a first-order language where the terms (the objects which can be quantified over) are those of the λ-calculus extended with logical constants, as discussed above. The language of wff has two predicates, P for "is a proposition", and T for "is a true proposition". Clearly, this gives the formal means for axiomatising the behaviour of propositions and true propositions. For example, the informal discussion concerning the behaviour of the logical constant $\wedge$ can be formalised as follows:

> "given the propositions t, s, the conjunction of these, $t \wedge s$, is also a proposition":
>
> $$\mathrm{P}(t)\,\&\,\mathrm{P}(s) \rightarrow \mathrm{P}(t \wedge s)$$
>
> "if t, s are propositions that are true, then $t \wedge s$ will also be a true proposition":
>
> $$\mathrm{P}(t)\,\&\,\mathrm{P}(s) \rightarrow (\mathrm{T}(t \wedge s) \leftrightarrow (\mathrm{T}(t)\,\&\,\mathrm{T}(s)))$$

Axioms for T must always be restricted so that only terms that are propositions are considered.

The distinction between wff and terms can be taken to be akin to that between extension and intension in Montague semantics Montague, 1974; Dowty et al., 1981. In that theory, however, intensions are derived from extensions.[13] As a consequence, the equality of intensions is that of the extensions, so propositions will be equated if they are always true together, and properties will be equated if they hold of the same objects. This is in contrast to PT, where the intensions are basic. Propositions in the language of terms may have the same truth conditions when T is applied, but this does not force them to be the same proposition, so we might have:

$$\mathrm{T}(s) \leftrightarrow \mathrm{T}(t)$$

but that does not mean that the terms are equal:

$$s = t$$

Similarly, in the language of wff, properties may hold of the same terms, yet they may be distinct. The λ-equality of terms is thus weaker than the notion of logical equivalence obtained when considering truth conditions in the meta-language.

A useful parallel can be drawn between PT and Frege's notions of *sense* and *reference*. We can take a proposition in the language of terms as corresponding to the *sense* of a statement. The *referents* of statements can be thought of as truth values, or truth conditions, which are obtained in the language of wff by applying T to the proposition.

It is possible to give a proof theory for the language of terms without recourse to a formal meta-theory like the language of wff Hindley and Seldin, 1986. This might not be so useful for the semantics of natural language, as the resultant theory would not explicitly capture the extensional level.

2.2 Formal Theory

The following presents a formalisation of the languages of terms and wff, together with the axioms that provide the closure conditions for P and T.

Language of terms

[13] In Montague's intensional logic IL, an intension is given by applying the operator ^ to an extension.

Basic Vocabulary:

Individual variables:	$x, y, z, \ldots$
Individual constants:	$c, d, e, \ldots$
Logical constants:	$\vee, \wedge, \neg, \Rightarrow, \Xi, \Theta, \approx$

Inductive Definition of Terms:

(i) Every variable or constant is a term.
(ii) If t is a term and x is a variable then $\lambda x.t$ is a term.
(iii) If t and t' are terms then $t(t')$ is a term.

The theory is governed by the following axioms:

Axioms of The $\lambda\beta$–Calculus

$$\begin{aligned} \lambda x.t &= \lambda y.t[y/x] \ y \text{ not free in } t \\ (\lambda x.t)t' &= t[t'/x] \end{aligned}$$

This defines the equivalence of terms.

All variables and constants, including the logical constants, are terms. Composition of terms allows us to build all those expressions of interest, and all those that are not. We need the meta-language, the language of wff, to classify those terms of interest and to give their logical behaviour.

The Language of Wff

Inductive Definition of Wff:

(i) If t and s are terms then $s = t, \mathrm{P}(t), \mathrm{T}(t)$ are atomic wff.
(ii) If φ and φ' are wff then $\varphi \,\&\, \varphi', \varphi \,\mathrm{v}\, \varphi', \varphi \rightarrow \varphi'$, $\sim\varphi$ are wff.
(iii) If φ is a wff and x a variable then $\exists x\varphi$ and $\forall x\varphi$ are wff.

The closure conditions for propositionhood are given by the following axioms:

Axioms of Propositions

(i) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \wedge s)$
(ii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \vee s)$
(iii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \Rightarrow s)$
(iv) $\mathrm{P}(t) \rightarrow \mathrm{P}(\neg t)$
(v) $\forall x \mathrm{P}(t) \rightarrow \mathrm{P}(\Theta\lambda x.t)$
(vi) $\forall x \mathrm{P}(t) \rightarrow \mathrm{P}(\Xi\lambda x.t)$
(vii) $\mathrm{P}(s \approx t)$

Truth conditions can be given for those terms that are propositions:

Axioms of Truth

(i) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \wedge s) \leftrightarrow \mathrm{T}(t) \,\&\, \mathrm{T}(s))$
(ii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \vee s) \leftrightarrow \mathrm{T}(t) \,\mathrm{v}\, \mathrm{T}(s))$
(iii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \Rightarrow s) \leftrightarrow \mathrm{T}(t) \rightarrow \mathrm{T}(s))$
(iv) $\mathrm{P}(t) \rightarrow (\mathrm{T}(\neg t) \leftrightarrow \ \sim\mathrm{T}(t))$
(v) $\forall x \mathrm{P}(t) \rightarrow (\mathrm{T}(\Theta\lambda x.t) \leftrightarrow \forall x \mathrm{T}(t))$
(vi) $\forall x \mathrm{P}(t) \rightarrow (\mathrm{T}(\Xi\lambda x.t) \leftrightarrow \exists x \mathrm{T}(t))$
(vii) $\mathrm{T}(t \approx s) \leftrightarrow t = s$
(viii) $\mathrm{T}(t) \rightarrow \mathrm{P}(t)$

The last axiom states that only propositions may have truth conditions.

Note that the quantified propositions $\Theta\lambda x.t$, $\Xi\lambda x.t$ can be written as $\Theta x(t)$, $\Xi x(t)$, where the λ-abstraction is implicit.

The notions of n-place relations can be defined recursively:

(i) $Rel_0(t) \leftrightarrow \mathrm{P}(t)$

(ii) $Rel_n(\lambda x.t) \leftrightarrow Rel_{n-1}(t)$

We can write $Rel_1(t)$ as $\mathrm{Pty}(t)$.

As an illustration of how this theory addresses the paradoxes that can arise when self-reference is permitted, consider a predicate R whose extension is those predicates that do not apply to themselves. If such a predicate exists, then we can derive the paradoxical proposition:

$$RR \leftrightarrow \sim RR$$

In PT we can define a term corresponding to R as follows:

$$R =_{\mathrm{def}} \lambda p.\neg pp$$

From the definition of Pty and the Axiom of Propositions (iv) we can trivially prove that if we have a property p, then Rp is a proposition, that is:

$$\mathrm{Pty}(p) \rightarrow \mathrm{P}(Rp)$$

Thus, the Axioms of Truth can be applied to Rp if p is a property. The paradoxical proposition originally arose when considering RR. For it to occur in PT, the Axioms of Truth must apply to this term. For these axioms to apply, we must show RR is a proposition. This in turn requires that R is a property. This cannot be proven, as R does not form a proposition with arbitrary terms, only with properties. Thus the paradoxical proposition does not arise. This exemplifies how PT allows unproblematic instances of self predication, whilst avoiding the category mistake that gives rise to paradoxical propositions.

After giving some examples of how we might use PT for natural language semantics, I shall embody a part-whole structure within the language of terms, following the essence of Link's theory Link, 1991a.

2.3 natural language Examples

As an example of how PT can be used in the semantics of natural language, the sentence:

Every boy laughed.

could be represented by the term:

$$\Theta x(\mathrm{boy}'x \wedge \mathrm{laughed}'x)$$

This object is independent of any truth conditions. To find the truth conditions of the sentence, we must first show that the term representing it is a proposition, that is:

$$\mathrm{P}(\Theta x(\mathrm{boy}'x \wedge \mathrm{laughed}'x))$$

This is an expression in the language of wff. According to the axioms for P, this will hold if:

$$\forall x(\mathrm{P}(\mathrm{boy}'x) \,\&\, \mathrm{P}(\mathrm{laughed}'x))$$

If the sentence is a proposition, then its truth conditions are given by:

$$\mathrm{T}(\Theta x(\mathrm{boy}'x \wedge \mathrm{laughed}'x))$$

According to the axioms, this holds if and only if:

$$\forall x(\mathrm{T}(\mathrm{boy}'x)\,\&\,\mathrm{T}(\mathrm{laughed}'x))$$

Not all sentences will express propositions. As an example, the axioms should not allow the representation of:

This sentence is false.

to be a proposition, otherwise the theory would fall foul of the paradoxes.

Not all logical constants in the representations of sentences will be interpreted as logical connectives in the truth conditions. The sentence:

Mary believes that every boy laughed.

might be represented with the term:

$$\mathrm{believe}'(\Theta x(\mathrm{boy}'x \wedge \mathrm{laughed}'x))(\mathrm{mary}')$$

If this is a proposition, then in its truth conditions T will not apply to "every boy laughed":

$$\mathrm{T}(\mathrm{believe}'(\Theta x(\mathrm{boy}'x \wedge \mathrm{laughed}'x))(\mathrm{mary}'))$$

This corresponds to the idea that the object of a believe is an intensional proposition, not a truth value, or set of possible worlds.

3 Mereology in PT

This section is concerned with axiomatising a mereological (part-whole) structure over the natural language denotable terms. It is intended to give a general treatment of part-whole relations which seems to be required for treatments of both mass terms and plurals. It is not necessarily a physical mereology, counter to the nominalistic bent of earlier presentations of mereology Leonard and Goodman, 1940. A model for this theory is given in the appendix.

To represent sentences involving mass terms and plurals in PT, we can add terms and axioms for a summation, a supremum operator, and a part-of relation. The axioms will resemble those of a Boolean algebra over a subclass of terms. In this formalisation, the axioms will be too weak to prove that the suprema of empty properties are equated. This will allow non-denoting definite descriptors to be distinguished.

Although there is no restriction on term formation in PT, it does seem appropriate to have such a structure only amongst those terms which can be referred to by natural language definite descriptors. This can be achieved by restricting the scope of the relevant axioms. To have a Boolean structure on all terms is an unmotivated and unnecessary strengthening of the theory.[14]

Lønning, in his consideration of the logical complexity of a representation with plural terms, notes that there may be some objects which cannot be denoted by natural language nominals, yet which cannot be excluded by consideration of the representation language in isolation. A formal theory of nominals (plurals or mass terms) can be said to be *complete* (in the lattice-theoretic sense) if any arbitrary

[14] An additional reason for having only a subclass of terms in a Boolean structure becomes apparent when considering the model theory of PT. A model of PT can be constructed using a lattice theoretic model for the λ-terms Turner, 1990. It is convenient to use the ordering and join in this lattice to model the summation and supremum operators. If, however, we give a Boolean structure to all terms, it is not possible to do this: no model of the λ-calculus could then be constructed.

collection of terms has a supremum (as opposed to just any finitely *denotable* collection having a supremum). This gives rise to a second-order logic which cannot be given a general model Orey, 1959; Lønning, 1989. However, it is not clear whether natural language allows or requires quantification over arbitrary sums that cannot be referred to directly by natural language.

Link only allows those sets that are the extensions of natural language derived properties to have suprema. This is called *definable completeness* Lønning, 1989; Link, 1991a. Lønning examines other options. In the theory presented in this section, set-like objects are represented by properties. Quantification over 'sets' is then taken to be quantification over the properties in the representation language. These properties are by their nature definable. Therefore, you cannot quantify over arbitrary sets within PT. This automatically leads to definable completeness of the language of representation with respect to its model. Further, to make sense of the semantics of nominals in this theory, the quantifiers contained in the representation of natural language sentences are constrained to range over only those terms which represent natural language nominals.

In summary, the following theory axiomatises a mereology as a Boolean algebraic-like structure, which will not be provably atomic, and will not provably contain a bottom element. The theory is intended to capture the structure that appears to underlie the referents of natural language nominal expressions.

Language of mereological terms

To the basic vocabulary of terms is added:

Predicative constant:	δ
Operative constants:	$\oplus, \otimes, \sigma$

It is intended that δ will hold of natural language denotable terms. The term σ is the supremum operator, and the term $\oplus$ is the summation operator. Within the domain of denotables, $\otimes$ will be its dual.

The language of wff remains unchanged from basic PT. The theory is governed by the axioms of the $\lambda\beta$–calculus as before, together with the previous axioms for propositions P and truth T.

Further Axioms For a Mereology in PT

The term $\oplus$ acts as a summation operator. The order in which the terms are combined is irrelevant, so the operator is both symmetric and associative:

Axiom 3.1 *Symmetry.*

$$a \oplus b = b \oplus a$$

Axiom 3.2 *Associativity.*

$$a \oplus (b \oplus c) = (a \oplus b) \oplus c$$

Combining a term with itself will result in nothing new:

Axiom 3.3 *Idempotence.*

$$a \oplus a = a$$

An ordering can be defined in terms of $\oplus$. This simplifies the expression of some of the axioms:

Definition 3.1 *Definable ordering.*

$$t \leq s =_{\text{def}} t \oplus s = s$$

An internal form of the ordering can also be defined:

Definition 3.2 *Internal ordering.*

$$t \ll s =_{\text{def}} t \oplus s \approx s$$

The first ordering is an expression in the language of wff, as such it is a proposition. The second, internal ordering is an expression in the language of terms. As such, it can be used directly in the semantic representation of sentences.

The terms that are of interest are those which correspond to the extensions of referring descriptions. The predicative constant δ is a property which holds of any such natural language denotable term:

Axiom 3.4 *Internal notion of denotable.*

$$\mathrm{P}(\delta t)$$

As syntactic sugar, the predicate Δ can be defined in the language of wff which holds of natural language denotables:

Definition 3.3 *A term is* denotable*, as an external wff, iff it is true that it is denotable as an internal proposition.*

$$\Delta t =_{\text{def}} \mathrm{T}(\delta t)$$

The sum of two denotables will itself be denotable:

Axiom 3.5 *The sum of denotables is a denotable.*

$$\Delta a \,\&\, \Delta b \rightarrow \Delta(a \oplus b)$$

We want to be able to form the suprema of classes of denotables. The term σ is intended to do this. It is not restricted to forming suprema of denotables, although we are only interested in its behaviour with denotables. The following axiom is indifferent to whether there is a bottom element, and to whether such an element is a denotable. It is also indifferent as to what a 'non-denoting' definite descriptor denotes: the axiom is deliberately too weak to justify saying that a 'non-denoting' definite descriptor is a denotable. If a property has a mixed extension, this axiom remains silent as to whether the supremum of such a property is a denotable. In effect, σ is a generalisation of $\oplus$.

Axiom 3.6 *If some property has an extension, and it is contained in the domain of denotables, then its supremum is also in the domain of denotables.*

$$\forall p((\mathrm{Pty}(p) \,\&\, \forall x(\mathrm{T}(px) \rightarrow \Delta(x)) \,\&\, \exists x(\mathrm{T}px)) \rightarrow \Delta(\sigma xpx))$$

This corresponds to a notion of completeness in the domain of denotables. As PT naturally has a Fregean interpretation of a set as a collection of objects that form the extension of some property, then this completeness is only of the *definable* sets: this axiom thus expresses the notion of definable completeness with bounded quantification.

My approach to the so-called 'non-denoting' definite descriptors is to deliberately adopt weak axioms that do not indicate their behaviour. In particular, the result of attributing a natural language property to a 'non-existent' term should not be a statement that is true, or false, it should be a term that cannot be proven to have either truth value. As propositions have truth values, this suggests that the result of such an attribution should not be a proposition. This effect can be achieved by having the denotation of 'properties' that arise in the representation of natural language restricted to a class of terms that only form propositions when attributed to terms that are provably denotable. I shall adopt a weaker view which defines a class of terms that cannot be *proven* to produce propositions when attributed to 'non-denoting' terms (or rather, terms that are not provably in the class of denotables):

Definition 3.4 *A property of denotables produces a proposition, given some denotable.*

$$\mathrm{Pty}_\Delta(p) =_{\mathrm{def}} \forall x(\Delta(x) \to \mathrm{P}(px))$$

We can define forms of the quantifiers that are restricted to denotable terms:

Definition 3.5

$$\forall_\Delta x\varphi =_{\mathrm{def}} \forall x(\Delta x \to \varphi)$$

Definition 3.6

$$\exists_\Delta x\varphi =_{\mathrm{def}} \exists x(\Delta x \,\&\, \varphi)$$

Restricting ourselves to these quantifiers, we achieve the effect of a free logic, where free variables range over all terms, and quantified variables range over denoting terms. As before, these quantifiers can be given internal analogues, and a supremum operator which is restricted to denotables can also be defined:

Definition 3.7

$$\Theta_\delta x(t) =_{\mathrm{def}} \Theta x(\delta x \Rightarrow t)$$

Definition 3.8

$$\Xi_\delta x(t) =_{\mathrm{def}} \Xi x(\delta x \wedge t)$$

Definition 3.9

$$\sigma_\delta x(px) =_{\mathrm{def}} \sigma x(\delta x \wedge px)$$

If two denotables are not equal, then there is a denotable that is part of one, but not part of the other. The antecedent should be strengthened, as two denotables may be unequal, yet one be wholly contained in the other:

Axiom 3.7 *Different denotables have different parts.*

$$\forall_\Delta xy(x \not\leq y \to \exists_\Delta u(u \leq x \,\&\, u \not\leq y))$$

The term $\sigma_\delta xpx$ is intended to form the fusion of all denotables having the property p. Any denotable with the property p should be part of this term:

Axiom 3.8 *All denotables having a particular property must be a part of the supremum of denotables having that property.*

$$\forall p \forall_\Delta y((\mathrm{Pty}_\Delta(p) \,\&\, \mathrm{T}(py)) \to y \leq \sigma_\delta xpx)$$

I shall illustrate the need for the following axiom by way of example. If we consider the denotable substance "cold mud", any bit of cold mud will be part of the fusion of mud. If we take the fusion of bits of cold mud, it will also be part of the fusion of mud:

Axiom 3.9 *If all denotables having property p are part of some other denotable y, then the supremum of denotables having p must also be a part of y.*

$$\forall p \forall_\Delta y((\mathrm{Pty}_\Delta(p) \mathbin{\&} \forall_\Delta x(\mathrm{T}(px) \to x \leq y)) \to \sigma_\delta xpx \leq y)$$

The intuition behind the next axiom is that different collections have different suprema. There is a complication, as there is no reference to atoms in this theory: (i) we cannot consider a unique way of dividing a denotable into its minimal parts; (ii) different collections of denotables may have the same supremum if they *cover* the same denotables; further, (iii) we cannot guarantee to pick denotables that are part of a denotable with the required property. However, in this final case, the denotable will have a part which has the required property:

Axiom 3.10 *Different portions of denotables have different suprema: something that is part of the supremum of a property must be part of something with that property, or have a part with that property.*

$$\forall p \forall_\Delta u((\mathrm{Pty}_\Delta(p) \mathbin{\&} u \leq \sigma_\delta xpx) \to (\exists_\Delta z(pz \mathbin{\&} u \leq z) \mathbin{\mathrm{v}} \exists_\Delta z(pz \mathbin{\&} z \leq u)))$$

Obviously, this is not much use if we have uncountable parts, but that would always cause problems.

It is possible to define an operator * which turns a property into a cumulative property Lønning, 1989; Link, 1991a:

Definition 3.10

$$^* =_{\mathrm{def}} \lambda p \lambda t(t \approx \sigma x(px \wedge x \ll t))$$

There is no obvious role for a distributive operator Lønning, 1989; Link, 1991a in this theory as it stands, as there are no atoms to distribute to. I will indicate how distributive behaviour can be regained in Section 4.

The denotable of which all other denotables are a part (top) can be given by:

Definition 3.11 *The top element $\top$ of the denotables is the supremum of all denotable terms.*

$$\top =_{\mathrm{def}} \sigma_\delta x(x \approx x)$$

Complement can also be defined:

Definition 3.12 *The complement $\overline{s}$ of the supremum s of denotables having property p is the supremum of denotables not having property p.*

$$\overline{\sigma_\delta x \varphi} =_{\mathrm{def}} \sigma_\delta x(\neg\varphi)$$

A bottom element $\perp$ can be defined as $\overline{\top}$, but it can not be proven that $\Delta\perp$.

Amongst the denotables, $\otimes$ is the dual of $\oplus$. This will be used in the final theory to model disjunction:

Axiom 3.11 *Within the domain of denotables, $\otimes$ is the dual of $\oplus$.*

$$\Delta a \mathbin{\&} \Delta b \to \overline{a \oplus b} = \overline{a} \otimes \overline{b}$$

I give the duality by way of an axiom, although it is possible to make both $\otimes$ and $\oplus$ definitional using the following equalities:

$$\begin{aligned} \sigma_\delta xpx \oplus \sigma_\delta xqx &= \sigma_\delta x(px \vee qx) \\ \sigma_\delta xpx \otimes \sigma_\delta xqx &= \sigma_\delta x(px \wedge qx) \end{aligned}$$

noting that any denotable a can be given as the supremum $\sigma_\delta x(x \approx a)$. These equalities are a consequence of the axioms given. If they are taken as definitional, then the axioms governing $\oplus$ (and $\otimes$) are satisfied.

To implement the formal semantics of natural language in a Montague style using PT, we can define types for the representations of syntactic categories. This follows an existing account Turner, 1990; Turner, 1992, where types are predicates in the language of wff, except that the types here use properties of denotables in place of plain properties:

Definition 3.13

$$Det_\Delta(f) =_{\text{def}} \forall x(\text{Pty}_\Delta(x) \rightarrow Quant_\Delta(fx))$$

Definition 3.14

$$Quant_\Delta(f) =_{\text{def}} \forall x(\text{Pty}_\Delta(x) \rightarrow \text{P}(fx))$$

These correspond to the syntactic categories of determiner and quantifier.

The function space types can be defined with:

Definition 3.15

$$(P \Longrightarrow Q)(f) =_{\text{def}} \forall x(P(x) \rightarrow Q(fx))$$

To have a conventional lattice-theoretic treatment of plurals would just require the addition that the denotable terms are founded on atoms. However, it is possible to treat count terms without recourse to atoms. For example, we could say that a singular property—corresponding to a singular count term—can hold of a term only if it does not hold of any proper part of that term. A plural property—corresponding to a plural count term—would then be the collective form of a singular property.

These axioms are no stronger than those for an atomic theory of plurals in PT Fox, 1993. As this is the case, a model of PT extended with an atomic Boolean algebra can also be used to show consistency of this potentially atomless theory.

In the next section, I show how these roles can be used to obtain the correct distributive behaviour for mass terms without assuming homogeneous extensions, and for plurals without assuming atomicity.

4 The Final Theory

We are now have most of the formal machinery in place to develop the semantic theory proper. Just to recap, this theory is intended to: (i) allow apparently coreferring items to corefer; (ii) control the distribution of properties into inhomogeneous terms; and, (iii) treat non-denoting descriptors as giving rise to a category mistake.

I will only examine sentences with intransitive verbs and unconjoined quantified noun phrases in detail. Coverage of transitive and ditransitive verbs would require some additional effort not least in empirically determining the acceptable readings. This theory produces only non-generic interpretations of natural language sentences.

First, the representations of natural language sentences used in this section are introduced. For clarity, these representations use determiners such as "all" and "every" that are not part of the formal theory so far.[15]

This is followed by rules that type these representations so that, if appropriate, they can be proven to be propositions, and hence carriers of truth values. Axioms are then presented which give the truth conditions of sentences.

[15] It would be fairly trivial to either add them to the theory, or to use logical determiners where appropriate.

The rest of the section is concerned with strengthening the theory in order to control distributive inferences using property modifiers, and to obtain other appropriate entailment patterns.

I shall not examine conjoined nouns and noun phrases in this semantic theory. This is not because it is impossible to do so, but because it avoids further combinatorial complexity, which would make central aspects of the theory harder to grasp.

I shall assume that we do not want to rule out the possibility that objects may be equated with a fusion of substance(s), or that 'different' objects, and substances—objects referred to by the suprema of different properties—may have the same extension, in particular, that compound substances like "muddy water" may have the same extension as their constituent substances, "mud and water".

4.1 Representations of Sentences

This paper assumes that sentences like:

The man dies.

are represented by terms of the form:

$$\text{the}'(\text{man}')(\text{dies}')$$

where the$'$ is a generalised quantifier that requires two properties.

Adjectives take the noun they modify as an argument, so:

The tall man dies.

is represented by:

$$\text{the}'(\text{tall}'\text{man}')(\text{dies}')$$

Adjectives and nouns appearing after the copula are treated in the same manner as verbs:

The bull is black.
The puddle is water.

being represented by:

$$\text{the}'(\text{bull}')(\text{black}')$$
$$\text{the}'(\text{puddle}')(\text{water}')$$

respectively.

As mentioned, this theory does not treat conjoined (and disjoined) nouns and noun phrases directly. However, one of the later examples does involve a disjunction of adjectives. Conjunction (disjunction) can be represented by $\hat{\oplus}$ ($\hat{\otimes}$, respectively), regardless of the conjoined (disjoined, respectively) categories. These operators are new terms in the language. Clearly, $\hat{\oplus}, \hat{\otimes}$ can be axiomatised much as $\oplus, \otimes$. The difference being that $\hat{\oplus}, \hat{\otimes}$ are atomic, and only conjoin finite sets of terms[16] It is intended that they represent generalised conjunction (disjunction) between arbitrary categories.

Assuming that there is no ellipsis, the sentences:

The drunk and alcoholic died.
The blue and red bike disappeared.
All phosphorus is red or white.

[16]They form an atomic Boolean algebra, rather than a complete Boolean algebra. The additions required to the model given in the appendix are fairly trivial.

can have readings represented by:

$$\text{the}'(\text{drunk}' \oplus \text{alcoholic}')(\text{died}')$$
$$\text{the}'((\text{blue}' \oplus \text{red}')\text{bike}'))(\text{disappeared}')$$
$$\text{all}'(\text{phosphorus}')(\text{red}' \otimes \text{white}')$$

For completeness, closure conditions for arbitrary conjoined and disjoined semantic categories are presented.

As we are interested in the truth conditions of the terms representing sentences, we should first be able to prove that they are terms which are capable of having truth values. That is, we should be able to prove that they are propositions. To do this, we need to type the semantics of the categories. The simplicity of the semantics proposed requires terms to belong to more than one type. This is possible in PT as it is only weakly typed.

4.2 Types

In principle, we could give arbitrary types to terms representing the objects from the various syntactic categories, as long as the representations of well formed sentences were propositions. However, for clarity, types will be used that closely mirror Montague style semantics, as in Turner, 1990; Turner, 1992. Thus, although the proposal above is to represent nouns with terms that do not have any arguments, when the truth conditions are given, they will behave as predicate-like objects, as is usual in a Montague-style treatment:

Lemma 4.1 *The denotations of nouns are of the type* Pty_{Δ}.

Using the same motivation, the type corresponding to the category of the determiners will be akin to that used in more conventional representations (restricted to the denotables). This will result in noun phrase being represented by quantifiers (of denotables). However, the definite descriptor "the" must be given a more complex type, compatible with the proposed treatment of non-denoting terms:

Lemma 4.2 *Except for the definite descriptor, the interpretations of determiners are of the type* Det_{Δ}.

The type of the definite descriptor "the" should be restricted so that it conforms to the goal of not being able to prove propositionhood of a sentence when it contains a definite descriptor which "fails to denote". Thus the denotation of "the" should only form a quantifier with a property of denotables which has an extension in the denotable terms:

Lemma 4.3 *The interpretation of the definite descriptor is of the type:*

$$\forall p((\text{Pty}_{\Delta}(p) \,\&\, \exists_{\Delta} x(\text{T}(px))) \rightarrow \mathit{Quant}_{\Delta}(\text{the}'p))$$

If we put aside the use of property modifiers for the moment, verb phrases will be properties of denotables. This typing ignores Strawson's and Lasersohn's observations on felicitous use of non-denoting noun phrases Strawson, 1964; Lasersohn, 1993. Besides copula phrases, the only verb phrases I shall consider are intransitive verbs. These will be represented by terms of the same type as verb phrases. This means that the syntactic rule which allows an intransitive verb to be considered as a verb phrase has no effect on the semantic type.

Lemma 4.4 *Intransitive verbs are of the type* Pty_{Δ}.

Adjectives can modify nouns. In the semantics, an adjective should take a property (of denotables) and produce a new property (of denotables):

Lemma 4.5 *Adjectives are in the type* $\mathrm{Pty}_\Delta \Longrightarrow \mathrm{Pty}_\Delta$.

In the proposed representation, adjectives can also behave like a verb phrase, when they appear after the copula, so they can also be of the same type as intransitive verbs:

Lemma 4.6 *Adjectives are of type* Pty_Δ.

The two types that adjectives belong to correspond with the the conventional typing for predicative and subsective uses of adjectives Kamp, 1975. A more linguistically principled theory might give the copula the explicit function of modifying the type of adjectives and nouns to that of verb phrases.

This is sufficient to prove propositionhood of the representations of felicitous sentences without conjunction or disjunction. Other axioms are required to type the sums and products of terms. These will strengthen the closure conditions of types. The motivation is to mirror the syntactic closure conditions for conjoined and disjoined categories in a grammar. When we conjoin or disjoin two nouns, the result is a noun. As the representation of a noun, at some level, corresponds to a property of denotables, then if we form the sum or product of two properties of denotables, then the result will be a new property of denotables. This is expressed in the following two axioms:

Axiom 4.1 *The sum of two properties of denotables, is a property of denotables.*

$$(\mathrm{Pty}_\Delta(r) \mathbin{\&} \mathrm{Pty}_\Delta(w)) \rightarrow \mathrm{Pty}_\Delta(r \hat{\oplus} w)$$

Axiom 4.2 *The product of two properties of denotables, is a property of denotables.*

$$(\mathrm{Pty}_\Delta(r) \mathbin{\&} \mathrm{Pty}_\Delta(w)) \rightarrow \mathrm{Pty}_\Delta(r \hat{\otimes} w)$$

It will be useful to indicate how to derive the truth conditions of complex properties of denotables, applied to denotables. The extension of a conjunction (disjunction) of properties of denotables is the intersection (union, respectively) of the extensions of the conjoined (disjoined, respectively) properties:

Axiom 4.3 *The sum of properties of denotables, applied to a denotable, is true if the conjunction of those properties, applied to that denotable, is true.*

$$(\mathrm{Pty}_\Delta(r) \mathbin{\&} \mathrm{Pty}_\Delta(w) \mathbin{\&} \Delta(x)) \rightarrow (\mathrm{T}(r \hat{\oplus} w)x \leftrightarrow \mathrm{T}(rx) \mathbin{\&} \mathrm{T}(wx))$$

Axiom 4.4 *The product of properties of denotables, applied to a denotable, is true if the disjunction of those properties, applied to that denotable, is true.*

$$(\mathrm{Pty}_\Delta(r) \mathbin{\&} \mathrm{Pty}_\Delta(w) \mathbin{\&} \Delta(x)) \rightarrow (\mathrm{T}(r \hat{\otimes} w)x \leftrightarrow \mathrm{T}(rx) \mathrm{\,v\,} \mathrm{T}(wx))$$

The closure conditions for the conjunction and disjunction of terms of the type $Quant_\Delta$ are given next:

Axiom 4.5 *The sum of two quantifiers of denotables is a quantifier of denotables.*

$$(Quant_\Delta(f) \mathbin{\&} Quant_\Delta(g)) \rightarrow Quant_\Delta(f \hat{\oplus} g)$$

Axiom 4.6 *The product of two quantifiers of denotables is a quantifier of denotables.*

$$(Quant_\Delta(f) \mathbin{\&} Quant_\Delta(g)) \rightarrow Quant_\Delta(f \hat{\otimes} g)$$

Finally, the closure conditions on propositions are:

Axiom 4.7 *The sum of two propositions is a proposition.*

$$(\mathrm{P}(p) \,\&\, \mathrm{P}(q)) \rightarrow \mathrm{P}(p \hat{\oplus} q)$$

Axiom 4.8 *The product of two propositions is a proposition.*

$$(\mathrm{P}(p) \,\&\, \mathrm{P}(q)) \rightarrow \mathrm{P}(p \hat{\otimes} q)$$

The sums and products of propositions will correspond to the conjunctions and disjunctions of sentences. Sentential conjunction and disjunction will have the standard interpretations. This can be enforced with:

Axiom 4.9 *The sum of two propositions is a true proposition if both propositions are true.*[17]

$$(\mathrm{P}(p) \,\&\, \mathrm{P}(q)) \rightarrow (\mathrm{T}(p) \,\&\, \mathrm{T}(q) \leftrightarrow \mathrm{T}(p \hat{\oplus} q))$$

Axiom 4.10 *The product of two propositions is a true proposition if one of the factor propositions is true.*

$$(\mathrm{P}(p) \,\&\, \mathrm{P}(q)) \rightarrow ((\mathrm{T}(p) \,\mathrm{v}\, \mathrm{T}(q)) \leftrightarrow \mathrm{T}(p \hat{\otimes} q))$$

4.3 Truth Conditions

Were we not using property modifiers, plurals and mass terms, then the truth conditions of the representations of simple sentences—for example, of the form "every p q" and "some p q"—could reflect the more conventional interpretations of the quantifiers Turner, 1990; Turner, 1992. Thus we would have, for example:

$$\begin{array}{lcl} \mathrm{P}(\mathrm{every}'(p)(q)) & \rightarrow & (\mathrm{T}(\mathrm{every}'(p)(q)) \leftrightarrow \mathrm{T}(\Theta_\delta x(px \Rightarrow qx))) \\ \mathrm{P}(\mathrm{some}'(p)(q)) & \rightarrow & (\mathrm{T}(\mathrm{some}'(p)(q)) \leftrightarrow \mathrm{T}(\Xi_\delta x(px \wedge qx))) \end{array}$$

As we are using property modifiers, the truth conditions must be complicated somewhat. They conditions can be stated with the axioms that follow.

The determiner "every" gives rise to universal quantification over the denotable terms:

Axiom 4.11

$$\mathrm{P}(\mathrm{every}'(p)(q)) \rightarrow (\mathrm{T}(\mathrm{every}'(p)(q)) \leftrightarrow \mathrm{T}(\Theta_\delta x(px \Rightarrow qpx)))$$

Notice that the verb phrase and noun combine to form a modified property qp. The types of the terms will have to be fixed if the right hand side of the internal biconditional is to form a proposition independently of these axioms.

The determiner "some" in a sentence should give rise to quantification over collections in the truth conditions:

Axiom 4.12

$$\mathrm{P}(\mathrm{some}'(p)(q)) \rightarrow (\mathrm{T}(\mathrm{some}'(p)(q)) \leftrightarrow \mathrm{T}(\Xi_\delta x(px \wedge qpx)))$$

[17] It might be thought that this axiom, together with Axiom 4.3 will allow us to show "some man walked and talked" if "some man walked and some man talked". However, this would require an improper existential elimination.

These truth conditions implicitly require the notion of atomicity (or singularity), that is, q only ranges over singular individuals, not mass terms or plurals. This could be guaranteed by a semantic restriction in the truth conditions. Indeed, I define a semantic notion of singularity later. Here, however, I shall assume that any grammar used to obtain the representations has syntactically restricted the use of "every" to semantically singular terms.

The determiner "all" is more problematic: in the truth conditions of a sentence it should not give rise to universal quantification over collections, as this always forces a distributive reading. It must thus be treated more akin to the definite descriptor. I shall allow sentences like "all p q" to be true even if the extension of the property p is empty:

Axiom 4.13

$$\mathrm{P}(\mathrm{all}'(p)(q)) \rightarrow (\mathrm{T}(\mathrm{all}'(p)(q)) \leftrightarrow (\Delta(\sigma_\delta xpx) \rightarrow \mathrm{T}(qp(\sigma_\delta xpx))))$$

The definite descriptor gives rise to predication of the supremum:

Axiom 4.14

$$\mathrm{P}(\mathrm{the}'(p)(q)) \rightarrow (\mathrm{T}(\mathrm{the}'(p)(q)) \leftrightarrow \mathrm{T}(qp(\sigma_\delta xpx)))$$

In order to make the right-hand sides of the biconditionals in these axioms provably propositions, independently, then the verbs must be able to act as property modifiers:

Lemma 4.7 *The denotations of verbs are in the type* $\mathrm{Pty}_\Delta \Longrightarrow \mathrm{Pty}_\Delta$

As nouns can also act as verb-like terms when appearing after the copula, they can also be in this type:

Lemma 4.8 *The denotations of nouns are in the type* $\mathrm{Pty}_\Delta \Longrightarrow \mathrm{Pty}_\Delta$

Adjectives are already of this type, so no additional lemma is required to type them for occurrences after the copula.

If verb phrases had been typed independently of intransitive verbs, and the type changing function of the copula was made explicit, then this last lemma would become redundant.

4.4 Property Modifiers

Considering terms of the form $\mathrm{died}'\mathrm{men}'x$, as mentioned before, I shall take died' here to be the semantic equivalent of a subsective adjective. Although I use the word "adjective", I am referring to a semantic category that need not be limited to the representation of natural language adjectives. This is not new. For example, Hoepelman has suggested that nouns may, in the semantics, be treated as a special case of adjective Hoepelman, 1983.

It is fruitful to look at the kinds of behaviours defined in work on the semantics of adjectives, as it transpires that these behaviours are also useful in explaining how to obtain intuitively useful inferences in a semantic theory which uses property modifiers to represent Landman's notion of roles. Later in this section, property modifiers are given behaviours which might seem rather *ad hoc* in isolation. However, the suggested inferences have independent motivation from considerations of the semantics of natural language modifiers, and would already be required in a comprehensive semantics of natural language.

As PT is weakly typed, to some extent we can avoid deciding whether adjectives should be exclusively property modifiers, or properties.[18] There is no need for formal devices to change the type of a subsective adjective (and general property modifiers), such as operators or dummy predicates.

I shall assume that properties introduced in the representation of natural language are typically properties of denotables, and that property modifiers are properties of denotables that can also modify other properties to produce new properties.[19]

To motivate the formal definition of a property modifier: if we take "red" to be a property modifier, then it may produce a proposition when applied to some natural language denotable term. Given some other property, like "book", a new property is created, that of being a red book. The proposition will be true if that denotable is red. Unless extra conditions are added, a red book need not be red, but it is a book.

Definition 4.1 *A* property modifier r *is a property of denotables which, given a property of denotables p, forms a new property of denotables rp, and if that new property holds of some denotables, then the original property of denotables p also holds of it.*

$$\mathcal{PM}(r) =_{\text{def}} \text{Pty}_\Delta(r) \mathbin{\&} \forall p(\text{Pty}_\Delta(p) \rightarrow (\text{Pty}_\Delta(rp) \mathbin{\&} \forall_\Delta x(\text{T}(rpx) \rightarrow \text{T}(px))))$$

As modifiers (both semantic and syntactic) can be conjoined and disjoined, it is necessary to add typing rules to make $\mathcal{PM}$ closed under $\hat{\oplus}$ and $\hat{\otimes}$.

Axiom 4.15 *The sum of two property modifiers is a property modifier.*

$$(\mathcal{PM}(r) \mathbin{\&} \mathcal{PM}(w)) \rightarrow \mathcal{PM}(r \hat{\oplus} w)$$

Axiom 4.16 *The product of two property modifiers is a property modifier.*

$$(\mathcal{PM}(r) \mathbin{\&} \mathcal{PM}(w)) \rightarrow \mathcal{PM}(r \hat{\otimes} w)$$

I will not show how these conjoined and disjoined modifiers effect the truth conditions of propositions until Example 4.4, towards the end of this section.

Following the suggestion that some of the behaviours of adjectives are desirable for property modifiers, we can define Bunt's notions of distributive, collective, and homogeneous adjectives Bunt, 1979[20]:

Definition 4.2 *A term r is* cumulative *(a cumulative modifier) iff it is a property modifier, and when restricting a property (of denotables) p, if it holds of denotables x, y, it also holds of the sum $x \oplus y$.*

$$\mathcal{C}_{\mathcal{PM}}(r) =_{\text{def}}$$
$$\mathcal{PM}(r) \mathbin{\&} \forall p \forall_\Delta xy((\text{Pty}_\Delta(p) \mathbin{\&} \text{T}(rpx) \mathbin{\&} \text{T}(rpy)) \rightarrow \text{T}rp(x \oplus y))$$

[18] For a comparison of these two approaches (considering 'predicates' rather than 'properties') see Kamp, 1975.

[19] It might be fruitful to formalise a version of this theory where the property modifiers are constructed from properties using some operator. This has been suggested to me by Gennaro Chierchia. In such a theory, rather than represent "John is a strict judge" with something like (strict′judge′j'), where strict′ is both a property of denotables, and a property modifier, we would use (as′judge′strict′j'), where judge′ is only a property of denotable, and as′ is a function that turns such a property into a property modifier. I suspect that this would simplify some aspects of the theory, though perhaps at the cost of a unification with a treatment of natural language modifiers.

[20] Obviously, I assume the adjectives in question are subsective, and that they act as property modifiers.

This can be generalised to arbitrary (definable) suprema:

Definition 4.2′

$$\mathcal{C}_{\mathcal{PM}}(r) =_{\text{def}} \quad \mathcal{PM}(r) \,\&\, \forall pq((\text{Pty}_\Delta(p) \,\&\, \text{Pty}_\Delta(q) \,\&\, \forall_\Delta x(\text{T}(qx) \rightarrow \text{T}(rpx))) \rightarrow \text{T}(rp(\sigma_\delta xqx)))$$

The following notion of distributive property modifiers is central to this theory's treatment of distributive inferences into denotable terms. Essentially, distribution must be restricted to the appropriate nice parts of a term. If r is distributive property modifier and p is a property, then if we predicate rp of a denotable x the appropriate nice parts of x to distribute to are those that have the property p. In general, we must not assume that r itself distributes to these parts, but allow rp to distribute (although for some pairs r, p, if rp holds, then we may infer r holds).

Definition 4.3 *A term r is* distributive *(a distributive modifier) iff it is a property modifier, and when restricting any property (of denotables) p, if it holds of denotables x, it also holds of any part y of the denotables x, which is also in the extension of p.*

$$\mathcal{D}_{\mathcal{PM}}(r) =_{\text{def}} \mathcal{PM}(r) \,\&\, \forall p \forall_\Delta xy((\text{Pty}_\Delta(p) \,\&\, \text{T}(rpx) \,\&\, \text{T}(py) \,\&\, y \leq x) \rightarrow \text{T}rp(y))$$

If we say "the water is liquid", and take the truth conditions of this to be interpreted as:

$$\text{T}(\text{liquid}'\text{water}'(\sigma x \text{water}' x))$$

then, assuming that liquid′ is distributive, any part of $\sigma x \text{water}' x$ that is water will be liquid water.

An homogeneous modifier is one that is both cumulative and distributive[21]:

Definition 4.4 *A term r is* homogeneous *(an homogeneous modifier) iff it is cumulative and distributive.*

$$\mathcal{H}_{\mathcal{PM}}(r) =_{\text{def}} \mathcal{C}_{\mathcal{PM}}(r) \,\&\, \mathcal{D}_{\mathcal{PM}}(r)$$

It seems to be the case that whenever the representation of a word is a cumulative property modifier, as defined above, it is typically a cumulative property of denotables, in the following sense:

Definition 4.5 *A strongly cumulative term is a cumulative property of denotables.*

$$\mathcal{C}_S(s) =_{\text{def}} \text{Pty}_\Delta(s) \,\&\, \forall_\Delta t(\text{T}(st) \leftrightarrow t = \sigma_\delta x(sx \wedge x \ll t))$$

Typically, those words that are considered to be homogeneous will be homogeneous modifiers, and cumulative properties:

Definition 4.6 *A strongly homogeneous term is an homogeneous modifier, and a strongly cumulative property.*

$$\mathcal{H}_S(S) =_{\text{def}} \mathcal{H}_{\mathcal{PM}}(s) \,\&\, \mathcal{C}_S(s)$$

[21] Note that these distributive and cumulative notions are different from those given by * and $^{\text{D}}$ Link, 1991a; Lønning, 1989, which produce collective and distributive properties respectively.

4.5 Singulars and Plurals

In the domain of count nouns, plurals are typically strongly homogeneous. We may wish to relate the interpretation of a plural to that of its syntactic singular. A singular term is not homogeneous. This result can be achieved as follows:

Definition 4.7 *A singular is a property of denotables which, if it holds of a denotable, does not hold of any of its parts.*

$$Sing(s) =_{\text{def}} \text{Pty}_\Delta(s) \,\&\, \forall_\Delta x(\text{T}(px) \rightarrow \;\sim\exists y(y \leq x \,\&\, \text{T}(py) \,\&\, y \neq x))$$

If s is a singular property, we can represent its (improper) plural form with ${}^\pi s$.[22] The intention of the next axiom is to state that any supremum of terms that have the singular property s will have the plural property ${}^\pi s$:

Axiom 4.17 *If some denotables each have the singular property p, then the supremum of those denotables has the plural property ${}^\pi p$.*

$$\forall p(Sing(p) \rightarrow \forall q(\text{Pty}_\Delta(q) \,\&\, \forall_\Delta x(\text{T}(qx) \rightarrow \text{T}(px)) \rightarrow \text{T}({}^\pi p(\sigma xqx))))$$

This is a generalisation of the intuition that the sum of two terms with the singular property s have the plural property ${}^\pi s$:

$$\forall p(Sing(p) \,\&\, \text{T}(pa) \,\&\, \text{T}(pb) \rightarrow \text{T}({}^\pi p(a \oplus b)))$$

There must also be a means of relating terms in the extension of a pluralised property with those in the extension of the singular property:

Axiom 4.18 *A term in the extension of a pluralised property must be founded on terms that are in the extension of the singular property.*

$$\forall p(Sing(p) \rightarrow \forall_\Delta u(\text{T}({}^\pi pu) \rightarrow \exists q(\text{Pty}_\Delta q \,\&\, \forall x(\text{T}(qx) \rightarrow \text{T}(px)) \,\&\, u = \sigma_\delta xqx)))$$

This is effectively the converse of the previous axiom. Taking these two axioms together, a plural property holds of a term if and only if that term is the fusion of terms having the singular property.

The equivalence:

$$\sigma xpx = \sigma x^\pi px$$

can be proved.

If a plural property holds of a term, but does not hold of any proper part of that term, then it must be the case that the underlying singular property also holds of the term:

Axiom 4.19 *If a denotable has the property ${}^\pi p$, (where p is a singular property) and no proper part of it does, then it also has the property p.*

$$\forall p \forall_\Delta x((Sing(p) \,\&\, \text{T}({}^\pi px) \,\&\; \sim\exists_\Delta y(y \leq x \,\&\, y \neq x \,\&\, \text{T}(py))) \rightarrow \text{T}(px))$$

In this formal theory, we also wish to be able to have plural forms of property modifiers:

Axiom 4.20 *Property modifiers have plural forms.*

$$\forall p((\mathcal{PM}(p) \,\&\, Sing(p)) \rightarrow \mathcal{PM}({}^\pi p))$$

[22] The improper plural form subsumes the singular form: ${}^\pi p$ will hold of one or more ps, as opposed to the two or more with English plurals.

Typically, when a plural property modifier is modifying a singular property, and the resultant property holds of a term, then the singular property modified by the singular property modifier also holds of that term. This is formalised in the following axiom:

Axiom 4.21 *If x is in the extension of ${}^{\pi}pq$ and q is singular, then x is also in the extension of pq.*

$$\forall pq \forall_{\Delta} x((\mathcal{PM}(p) \,\&\, \mathit{Sing}(p) \,\&\, \mathrm{Pty}_{\Delta}(q) \,\&\, \mathit{Sing}(q) \,\&\, \mathrm{T}({}^{\pi}pqx)) \rightarrow \mathrm{T}(pqx))$$

This results in a useful inference, allowing the representation of "the men die" to entail the representation of "every man dies" (assuming there are some men). This will be elaborated in Example 4.1, below.

4.6 Examples and Further Axioms

Example 4.1 I shall demonstrate the use of these notions with the sentence "the men die". If there are some men (so that the sentence forms a proposition), from the application of T to its representation:

$$\mathrm{T}(\mathrm{die}'\mathrm{men}'(\sigma_{\delta} x \mathrm{men}' x))$$

we can derive the truth conditions for "every man dies":

$$\forall_{\Delta} x(\mathrm{T}(\mathrm{man}' x) \rightarrow \mathrm{T}(\mathrm{dies}'\mathrm{man}' x))$$

I will take the two terms, men$'$ and die$'$ to be strongly homogeneous property modifiers. Further, I will take it that men$' = {}^{\pi}$man$'$, and die$' = {}^{\pi}$dies$'$. From the assumption that die$'$ is a property modifier, we can infer:

$$\begin{aligned}&\mathrm{T}(\mathrm{die}'\mathrm{men}'(\sigma_{\delta} x \mathrm{men}' x)) \rightarrow \\ &\quad \forall_{\Delta} y(\mathrm{T}(\mathrm{men}' y) \,\&\, y \leq (\sigma_{\delta} x \mathrm{men}' x) \rightarrow \mathrm{T}(\mathrm{die}'\mathrm{men}' y))\end{aligned}$$

Given the homogeneous behaviour of men, and its relation to its singular, we can show:

$$\begin{aligned}&\mathrm{T}(\mathrm{die}'\mathrm{men}'(\sigma_{\delta} x \mathrm{man}' x)) \rightarrow \\ &\quad \forall_{\Delta} y(\mathrm{T}(\mathrm{man}' y) \,\&\, y \leq (\sigma_{\delta} x \mathrm{man}' x) \rightarrow \mathrm{T}(\mathrm{die}'\mathrm{man}' y))\end{aligned}$$

From the axioms for σ, this gives:

$$\mathrm{T}(\mathrm{die}'\mathrm{men}'(\sigma_{\delta} x \mathrm{man}' x)) \rightarrow \forall_{\Delta} y(\mathrm{T}(\mathrm{man}' y) \rightarrow \mathrm{T}(\mathrm{die}'\mathrm{man}' y))$$

From the fact that die$'$ is the plural of dies$'$, Axiom 4.21 allows us to infer that the internal consequent implies:

$$\mathrm{T}(\mathrm{dies}'\mathrm{man}' y)$$

Thus:

$$\mathrm{T}(\mathrm{die}'\mathrm{men}'(\sigma_{\delta} x \mathrm{men}' x)) \rightarrow \forall_{\Delta} y(\mathrm{T}(\mathrm{man}' y) \rightarrow \mathrm{T}(\mathrm{dies}'\mathrm{man}' y))$$

This example shows that even with a semantic theory intended to cover some awkward examples, our intuitions concerning simpler cases are supported, if rather indirectly. •

It is possible to define notions corresponding to transparent, or predicative adjectives, like those presented by Bunt and Roeper Bunt, 1985; Roeper, 1983. The behaviour defined is required to cope with Bunt's "wet puddle" argument, which is formalised in the next example.

Definition 4.8 *A term r is* transparent *(a transparent modifier) with respect to a property (of denotables) p, iff it is a property modifier, and when restricting p, if it holds of denotables x, it also holds of x by itself. Further, if r and p hold of x, by themselves, then rp holds of x.*

$$\mathcal{T}_p(r) =_{\text{def}} \mathcal{PM}(r) \,\&\, \text{Pty}_\Delta(p) \,\&\, \forall_\Delta x(\text{T}rpx \leftrightarrow (\text{T}rx \,\&\, \text{T}px))$$

This is rather like Hoepelman's notion of *strongly predicative adjectives*, except that here transparency is indexed to *particular* properties of denotables, whereas strongly predicative adjectives are transparent with respect to *all* properties (of denotables). His notion of weakly predicative adjectives cannot be expressed in this theory as it stands, as there is no formal notion of polar opposites (tall v. short, for example).[23] We may wish to maintain that all property modifiers are transparent with respect to themselves, echoing one of Landman's axioms, which can be paraphrased as "John as a judge is a judge" Landman, 1989:

$$\forall p(\mathcal{PM}(p) \rightarrow \mathcal{T}_p(p))$$

This does not cause the collapse of compound property modifiers, which might be useful in a theory of adjectives: Hoepelman, for example, would use tall′tall′man′x to indicate a man who is tall for a tall man. If, however, we take chessplayer′ to be a transparent property modifier, it would cause the collapse of chessplayer′chessplayer′x to just chessplayer′x. Hoepelman would prefer to use this to indicate chessplayers who are relatively good at chess, in a body of other chessplayers Hoepelman, 1983.

We are now in a position to consider some more examples. The next two show how the theory can address Bunt's *desiderata* for a theory of mass terms Bunt, 1985.

Example 4.2 Considering the argument:

All water is wet
The puddle is water
∴The puddle is wet

Assuming that there is a puddle, the truth conditions of the three sentences are given by:

$$\text{T}(\Theta_\delta x(\text{water}'x \Rightarrow \text{wet}'\text{water}'x))$$
$$\text{T}(\text{water}'\text{puddle}'(\sigma_\delta x\text{puddle}'x))$$
$$\text{T}(\text{wet}'\text{puddle}'(\sigma_\delta x\text{puddle}'x))$$

After applying the axioms of truth, the argument becomes:

$$\forall_\Delta x(\text{T}(\text{water}'x) \rightarrow \text{T}(\text{wet}'\text{water}'x))$$
$$\text{T}(\text{water}'\text{puddle}'(\sigma_\delta x\text{puddle}'x))$$
$$\therefore \text{T}(\text{wet}'\text{puddle}'(\sigma_\delta x\text{puddle}'x))$$

The terms wet′ and water′ can be taken to be transparent, with respect to each other and puddle′.[24] From the transparency of water′ with puddle′, and the second premise, we have:

$$\text{T}(\text{water}'(\sigma_\delta x\text{puddle}'x))$$

[23] Hoepelman suggests we might take "red" to be weakly predicative: a red tomato may be red compared to the existent tomatoes, but we might not wish to class it as unadorned "red". It is weakly predicative as a red tomato is definitely not "unred", the polar opposite of "red", (green, blue, black, etc.), and a tomato that is truly red is a red tomato.

[24] Note that if we additionally take wet′ to be distributive, it does not mean that "wet" distributes to all parts. The distribution is only motivated when the distributive 'property' appears as a property modifier, so it can still only distribute to relevant parts.

and thus, from the first premise, we obtain:

$$\mathrm{T}(\mathrm{wet}'\mathrm{water}'(\sigma_\delta x \mathrm{puddle}'x))$$

From this and the transparency of wet′ with water′, we have:

$$\mathrm{T}(\mathrm{wet}'(\sigma_\delta x \mathrm{puddle}'x))$$

and from the transparency of wet′ with puddle′:

$$\mathrm{T}(\mathrm{wet}'\mathrm{puddle}'(\sigma_\delta x \mathrm{puddle}'x))$$

which is what was wanted. •

Example 4.3 Taking the sentence "all water is water", the truth conditions of its representation are given by:

$$\Delta(\sigma x \mathrm{water}'x) \rightarrow \mathrm{T}(\mathrm{water}'\mathrm{water}'(\sigma x \mathrm{water}'x))$$

Assuming that there is some water, the truth conditions of the sentence are dependent upon:

$$\mathrm{T}(\mathrm{water}'\mathrm{water}'(\sigma x \mathrm{water}'x))$$

From the transparency of water′, with respect to itself[25], and our assumption that it can also be treated as a property modifier, we have:

$$\forall_\Delta x(\mathrm{T}(\mathrm{water}'x) \leftrightarrow \mathrm{T}(\mathrm{water}'\mathrm{water}'x))$$

From the homogeneity of water′, and axioms for the supremum operator (assuming that there is some water), we can infer that:

$$\mathrm{T}(\mathrm{water}'(\sigma x \mathrm{water}'x))$$

Taking these two results together, we can infer:

$$\mathrm{T}(\mathrm{water}'\mathrm{water}'(\sigma x \mathrm{water}'x))$$

on the assumption that there is some water, thus we obtain the desired result (cancelling the assumption):

$$\Delta(\sigma x \mathrm{water}'x) \rightarrow \mathrm{T}(\mathrm{water}'\mathrm{water}'(\sigma x \mathrm{water}'x))$$

Thus "All water is water" is true. •

So the results that Bunt considers to be essential for a theory of mass terms are obtained.

Next, it is shown that the theory can cope with Roeper's "phosphorus" example, provided that the theory is strengthened to indicate how the disjunction (product) of property modifiers affects the truth conditions. Following this, I strengthen the behaviour of conjoined (summed) property modifiers.

Example 4.4 In the theory as it stands, the truth conditions of the sentence "all phosphorus is red or white" Roeper, 1983:

$$\mathrm{T}(\Theta_\delta x(\mathrm{phosphorus}'x \Rightarrow (\mathrm{red}'\hat{\otimes}\mathrm{white}')\mathrm{phosphorus}'x))$$

cannot be unpacked as far as we might like. To give a full elaboration of the truth conditions requires an additional axiom for the product of subsective adjectives.

[25] We cannot claim that it is transparent, period, as there is a counter example: a water meadow is not water.

Axiom 4.22 *A property of denotables p modified by the product of two property modifiers r, w holds of a denotable x, iff either (1) the property of denotables, modified by either property modifier, holds of that denotable; or (2) that denotable can be divided in two u, v, such that the property of denotables holds of one part u, when modified by the first property modifier, r, and the other part v, when modified by the second, w.*

$$\begin{aligned}&\forall rwp\forall_{\Delta}x(\mathcal{PM}(r)\,\&\,\mathcal{PM}(w)\,\&\,\mathrm{Pty}_{\Delta}p \rightarrow\\ &\qquad (\mathrm{T}((r \otimes w)px) \leftrightarrow (\ \mathrm{T}(rpx)\ \mathrm{v}\\ &\qquad\qquad \mathrm{T}(wpx)\ \mathrm{v}\\ &\qquad\qquad \exists_{\Delta}uv(x = u \oplus v\,\&\,\mathrm{T}(rpu)\,\&\,\mathrm{T}(wpv))))\end{aligned}$$

Thus, "all phosphorus is red or white" is true, on this narrow scope reading of the disjunction, if it is the case that any fusion of phosphorus is red; or it is white; or if it has two parts that are phosphorus, and one part is white, and the other is red. Note that this narrow scope reading subsumes the wide scope reading of the disjunction ("all phosphorus is red, or all phosphorus is white"). •

An apparent inadequacy of this axiom is that if there is some gold making up a ring and the gold is half white gold and half yellow gold, then, from axiom above we can infer:

The gold is white or yellow.

It should be noted that this conclusion does not subsume the wide scope reading:

The gold is white or the gold is yellow.

Even so, it might seem a slightly odd conclusion. We can argue that the narrow scope disjunction is fine, as it might be paraphrased:

The quantities of gold that make up the ring are white or yellow.

The reason we do not usually utter the disjunction is that, as a sentence (rather than a paraphrase of a conclusion), it is ambiguous since it subsumes the representation of the sentence:

The gold is white or the gold is yellow.

We can then appeal to pragmatic considerations that suggest we strive to be maximally informative, and hence are more likely to use the sentence:

The gold is white and yellow.

rather than:

The gold is white or yellow.

It can seem unsatisfactory when an appeal to pragmatics is made to gloss over some difficulties in an essentially formal theory. However, in this case some comfort can be drawn from the other uses of disjunction. In a Montague-style analysis, from:

Mary laughed and John talked.

we can infer:

Mary laughed or John talked.

even though we would be unlikely to utter the disjunction in place of the conjunction should it be the case that we know Mary laughed and John talked.

I think that we can say little about the case of conjoined property modifiers. If something is black and white, it may be acceptable to say that there are parts of it that are black, and parts that are white. However, in dissecting the object, these attributions of colour may be invalid.[26]

If we take the sentence:

John is an angry and hateful person.

it is not easy to contemplate the idea that there are necessarily parts of John which are angry, and other parts which are hateful. We could account for this sentence by assuming movement has occurred (in the syntax) from "John is an angry person and John is a hateful person". However, we may then posit movement in cases of exclusive properties, black and white, for example. These readings could be ruled-out by semantic considerations. In which case, we might also want the constraint that $(r\hat{\oplus}w)px$ can only be a proposition if rp, wp are exclusive properties.

With or without this constraint, I am fairly certain that we can have the following:

Axiom 4.23 *If a property of denotables p, modified by a property modifier r, holds of a denotable u, and modified by another property modifier w, holds of another denotable v, then the property of denotables, modified by the sum of the property modifiers $r\hat{\oplus}w$, holds of the (denotable) sum of denotables $u\hat{\oplus}v$.*[27]

$$\forall rwp\forall_{\Delta}uv((\mathcal{PM}(r)\,\&\,\mathcal{PM}(w)\,\&\,\mathrm{Pty}_{\Delta}p\,\&\,)\rightarrow \\ ((\mathrm{T}(rpu)\,\&\,\mathrm{T}(wpv))\rightarrow \mathrm{T}((r\hat{\oplus}w)p(u\oplus v))))$$

In the case when rp, wp are exclusive properties, we may also have the converse of this.

Axiom 4.24 *If two property modifiers w, r are exclusive, when modifying a property of denotables p, then if the sum $w\oplus r$ modifying p holds of a denotable x, then x is the sum of denotables u, v, where wp holds of u and rp holds of v.*[28]

$$\forall wrp(\mathcal{PM}w\,\&\,\mathcal{PM}r\,\&\,\mathrm{Pty}_{\Delta}p\,\&\,\sim\exists_{\Delta}x(\mathrm{T}(wpx)\,\&\,\mathrm{T}(rpx))\rightarrow \\ \forall_{\Delta}x(\mathrm{T}((w\hat{\oplus}r)px)\rightarrow\exists_{\Delta}uv(x=u\oplus v\,\&\,\mathrm{T}(wpu)\,\&\,\mathrm{T}(rpu))))$$

The next example shows how the theory allows contradictory properties to be attributed to objects, even when their extensions are equated. Although with the example chosen, we might prefer not to equate the extensions for philosophical reasons, it will serve as an illustration.

Example 4.5 The well-worn sentences:

The gold ring is new.
The gold is old.

[26] Confusion is increased with the uses of "black and white", for example, in "black and white television".

[27] It might be thought that from something like "some ball is a red ball and some ball is a white ball" we are able to infer "some ball is a red and white ball". However, this would be to assume that "some ball" is itself a denotable term, rather than something which gives rise to quantification over denotable terms.

[28] As it stands this axiom is too strong for plurals. From "the red and white ball p" we might infer that there is a red ball and there is a white ball". If we made explicit the analysis of distribution into syntactically conjoined terms, then this problem could be avoided.

become:

$$\text{the}'(\text{gold}'\text{ring}')(\text{new}')$$
$$\text{the}'(\text{gold}')(\text{old}')$$

Assuming there is some gold and a gold ring, the truth conditions of these terms are given by:

$$\text{T}(\text{new}'(\text{gold}'\text{ring}')(\sigma_\delta x(\text{gold}'\text{ring}')x))$$
$$\text{T}(\text{old}'\text{gold}'(\sigma_\delta x \text{gold}' x))$$

Even if the gold is realised by the gold ring:

$$\sigma_\delta x(\text{gold}'\text{ring}')x = \sigma_\delta x \text{gold}' x$$

there is no intrinsic contradiction in the truth conditions of these sentences. •

The final example shows how the theory copes with the comparative sentences which Landman uses to argue against a property modifier treatment of roles Landman, 1989.

Example 4.6 Take the comparative sentence:

The judge and the cleaner earn different incomes.

Essentially, its treatment in the theory rides on the meaning of "earn different incomes". In a strongly typed logic, it may be hard to see how "earn different incomes" can be expressed. Weakly typed logics like PT make it easier, the expression is just earn′(different′incomes′). We can represent the sentence as:

$$((\text{the}'\text{judge}')\hat{\oplus}(\text{the}'\text{cleaner}'))(\text{earn}'(\text{different}'\text{incomes}'))$$

where:

$$\begin{aligned} &\text{T}(((\text{the}'\text{judge}')\hat{\oplus}(\text{the}'\text{cleaner}'))(\text{earn}'(\text{different}'\text{incomes}'))) \leftrightarrow \\ &\quad \text{T}(\Xi_\delta ab(\quad \text{income}' a \wedge \text{income}' b \wedge \\ &\quad\quad \text{the}'(\text{judge}')(\text{earn}' a) \wedge \text{the}'(\text{cleaner}')(\text{earn}' b) \wedge \\ &\quad\quad \text{different}'({}^{\pi}\text{income}')(a \oplus b))) \end{aligned}$$

This meaning postulate should be generalised to cope with arbitrary noun phrases.

We can give a weak meaning postulate for "different incomes", along the lines:

$$\begin{aligned} &\forall_\Delta u(\text{T}({}^{\pi}\text{income}' u) \rightarrow \\ &\quad (\exists_\Delta ab(\text{T}(\text{income}' a) \,\&\, \text{T}(\text{income}' b) \,\&\, a \leq u \,\&\, b \leq u \,\&\, a \neq b) \rightarrow \\ &\quad\quad \text{T}(\text{different}'({}^{\pi}\text{income})u))) \end{aligned}$$

•

so that "different incomes" hold of something if it is a collection of incomes, and two of the incomes are not equal.

The treatment of some of the examples is perhaps rather indirect and involved. However, the main reason for giving the details is to show that the ideas developed can be used in compositional semantics. Clearly some additional effort must be expended to extend the coverage of this theory to transitive and ditransitive verbs, and to incorporate a dynamic component to produce an analysis of intermediate distribution and reciprocals as suggested by Schwarzschild Schwarzschild, 1990; Schwarzschild, 1992.

No treatment of committee-like objects, or proper names is offered here. It should be a simple matter to add Barker's account of collective nouns Barker, 1992.

5 Conclusions

If there are lessons to be learnt from this paper, they are that (i) care should be taken in assuming a particular ontology is empirically motivated just because it helps to cope with problems in some particular formal theory of natural language semantics. By way of illustration, a formal treatment for some plural and mass term phenomena can be provided without having to come to decisions on what perhaps are essentially philosophical, rather than semantic issues Parsons, 1975 concerning homogeneity and atomicity. Similarly, (ii) care should be taken in assuming that certain empirical phenomena, such as non-denoting definites, necessitate a move away from classical two-valued logic in the truth-conditional semantics of natural language.

The theory has a rather limited coverage. However, the manner in which the issues are addressed exemplifies an axiomatic approach to semantics. In essence, the methodology of devising formal semantics theories is not seen as providing some compositional mapping between language and a mathematical model. Rather, the central goal is seen as obtaining appropriate inferences within a tractable theory. If we seek to do this by way of model-theoretic semantics, then there is the constant risk of building a theory that is too strong (it might sanction undesirable, or unmotivated inferences) or two powerful (where there may be no tractable proof theory). In this axiomatic treatment, we are free to choose axioms for the representation language that sanction only those inferences which conform to our intuitions. The theory can remain incomplete with respect to issues for which we have no clear, theory independent intuitions.

The weak typing of PT helps simplify the task of producing a semantic theory, as terms can belong to more than one type. For example, one term may appear as both a property, and a property modifier. There is no strong hierarchy of types, which might require various type-lifting strategies over semantic terms.[29].

Strong typing is often used in natural language semantic theories to avoid the self-predication paradoxes, whereas PT avoids these paradoxes axiomatically, making the theory too weak to prove that paradoxes of self-predication are propositions. Incomplete axioms can also be used to address the category mistake apparent in predication of non-denoting terms and to proofs of the existence of atomic mass terms. The latter satisfies our intuitions that mass terms can be used regardless of one's theory of substances and matter. Further, this allows the effects of different ontological choices on the semantics of nominals to be explored within the one theory: the final theory neither forces nor prevents the formal equality of certain terms, like "the mud and the water" with "the muddy water".

If we use a first-order theory, such as PT, then we can guarantee that we have a semi-decidable proof theory. This means that it lends itself to implementation. As the behaviour of natural language representations are described directly as axioms in PT, rather than as restrictions on a model of the representation, a system that uses this theory for semantic representation can perform useful inferences. This is surely a desirable objective for formal semantics: not just to provide a symbolic representation of sentences, but to indicate how intuitively acceptable inferences can be performed, preferably in a computationally tractable framework.

The weak treatment of non-denoting terms is compatible with a proof-theoretic implementation of a natural language system. As an example, the failure to ascribe a truth value to the sentence "the present king of France is bald" can be mechanically demonstrated to be due to the non-existence of "the present king of France". Thus, in an implementation of a question answering system, the helpful answer:

[29]Perhaps to be distinguished from type-lifting as used in categorial grammars to account for ellipsis Dowty, 1988.

> There is no present king of France.

could be generated automatically, in response to the question:

> Is the present king of France bald?

Clearly, such an implementation would be dependent upon a suitable formal theory of questions and answers.

The treatment of existence presuppositions could be examined further, and may prove to be extendible to cover other examples of presupposition.

The paper does not address the contextual, dynamic effects that seem to affect intermediate distributive readings and reciprocals Schwarzschild, 1990; Schwarzschild, 1992. A strengthened PT can be used to embody Martin–Löf's type theory Martin-Löf, 1982; Martin-Löf, 1984 Turner, 1990, Chapter 5. This can, in turn, be used to model some of the dynamic aspects of natural language semantics Sundholm, 1989; Ranta, 1991; Davila-Perez, 1994. It then seems that there is scope for further work directed at treating dynamic effects in PT, using ideas from Martin–Löf's type theory Fox, 1994b; Fox, 1994a, which might be used to account for the contextual effects required to obtain a reductive analysis of intermediate distributive readings and reciprocals. A property modifier could indicate the relevant parts, where that property modifier is obtained anaphorically following Schwarzschild's suggestion. This would extend the application of property modifiers in the domain of plurals.

A A Model of PT with Boolean Terms

We can show that property theory with mereological terms (PT+Δ) is consistent if we can provide a model which verifies all of its axioms.

First of all we need a model for the λ-calculus. This can be used to build a model of PT. We then require the model of PT to be strengthened to satisfy the axioms for denotable terms. Link and Lønning have both effectively shown that their axioms, with atomicity, are satisfiable if the denotable domain is a (definably) complete atomic Boolean algebra Link, 1991a; Lønning, 1989. As my axioms for denotables are essentially a weaker version of Link's axioms in Link, 1991a then a Boolean algebra should verify them also.

We thus need a model of PT, where the natural language denotable terms belong to a complete Boolean algebra. The model presented for PT shall 'naturally' satisfy full completeness, as opposed to definable completeness. However, as we are only interested in showing that PT+Δ is consistent, then the model can be stronger than this theory: if we only need definable completeness, then it does not matter if we actually have full completeness in the model, nor does it matter that the Boolean algebra is atomic.

A.1 A Model of the λ-Calculus with Summed Terms

Following an existing approach Scott, 1973, we shall build a model of the λ-calculus from *domains* consisting of *complete lattices*. In the limit we have a domain D_∞ isomorphic to its own continuous function space, so $D_\infty \cong [D_\infty \longrightarrow D_\infty]$. We can define mappings $\Phi : D \longrightarrow [D \longrightarrow D]$ and $\Psi : [D \longrightarrow D] \longrightarrow D$.

Definition A.1 *A* Scott Model *is a triple $\mathcal{D} = \langle D, \Phi, \Psi \rangle$ with D a domain and Φ, Ψ as above.*

The terms of λ-calculus can be interpreted in such a structure relative to an assignment function g which assigns elements of D to variables, and interpretation function i which assigns elements of D to constants. The function $g[d/x]$ is the function g except that d is bound to x. Reference to $\mathcal{D}$ is dropped in the following. i is assumed to be fixed.

$$\begin{array}{rcl} \mathcal{I}[x]_g & = & g(x) \\ \mathcal{I}[c]_g & = & i(c) \\ \mathcal{I}[\lambda xt]_g & = & \Psi(\lambda d.\mathcal{I}[t]_{g[d/x]}) \\ \mathcal{I}[t(t')]_g & = & \Phi(\mathcal{I}[t]_g)(\mathcal{I}[t']_g) \end{array}$$

We want to be able to give a model of the λ-calculus extended with sums and products of terms. We can do this with:

$$\begin{array}{rcl} \mathcal{I}[t \oplus t']_g & = & \bigsqcup\{\mathcal{I}[t]_g, \mathcal{I}[t']_g\} \\ \mathcal{I}[t \otimes t']_g & = & \bigsqcap\{\mathcal{I}[t]_g, \mathcal{I}[t']_g\} \end{array}$$

A.2 A Model of PT+Δ

Following Aczel, 1980:

Definition A.2 *A model for PT shall be taken to be a Frege structure* $\mathcal{M} = \langle \mathcal{D}, T, P \rangle$ *where* $\mathcal{D}$ *is a model of the Lambda Calculus and*

$$\begin{array}{c} T : D \longrightarrow \{0, 1\} \\ P : D \longrightarrow \{0, 1\} \end{array}$$

Where T and P satisfy the structural requirements in Aczel, 1980.

The characteristic functions T and P provide the extensions of the truth predicate and the proposition predicate, respectively. The structural requirements they conform to verify the appropriate axioms of PT. For example, the function T characterises a subset of P. Thus the terms have a subclass consisting of terms that correspond to propositions. This subclass, in turn, has a subclass of terms corresponding to the true propositions.

The language of wff can now be given truth conditions.

$$\begin{array}{lll} \mathcal{M} \models_g s = t & \text{iff} & \mathcal{I}[t]_g = \mathcal{I}[s]_g \\ \mathcal{M} \models_g \mathrm{T}(t) & \text{iff} & T(\mathcal{I}[t]_g) = 1 \\ \mathcal{M} \models_g \mathrm{P}(t) & \text{iff} & P(\mathcal{I}[t]_g) = 1 \\ \mathcal{M} \models_g \varphi \,\&\, \psi & \text{iff} & \mathcal{M} \models_g \varphi \text{ and } \mathcal{M} \models_g \psi \\ \mathcal{M} \models_g \varphi \,\mathrm{v}\, \psi & \text{iff} & \mathcal{M} \models_g \varphi \text{ or } \mathcal{M} \models_g \psi \\ \mathcal{M} \models_g \varphi \rightarrow \psi & \text{iff} & \mathcal{M} \models_g \varphi \text{ implies } \mathcal{M} \models_g \psi \\ \mathcal{M} \models_g \sim\varphi & \text{iff} & \mathcal{M} \models_g \text{ not } \varphi \\ \mathcal{M} \models_g \forall x\varphi & \text{iff} & \text{for all } d \in D\ \mathcal{M} \models_{g[d/x]} \varphi \\ \mathcal{M} \models_g \forall x\varphi & \text{iff} & \text{for some } d \in D\ \mathcal{M} \models_{g[d/x]} \varphi \end{array}$$

A wff φ of PT is valid in a model $\mathcal{M}$ iff $\mathcal{M} \models_g \varphi$ for all assignment functions g.

For a model of PT+Δ we need a stronger base model than $\mathcal{M}$. We require those terms representing denotables to form a complete (atomic) Boolean algebra. Models of the λ-calculus, in general, do not possess these properties. This problem can be addressed by giving a substructure of $\mathcal{M}$ the desired properties, and letting the natural language denotable terms denote appropriate objects in this structure. Thus, denotable objects will form a sub-domain.

A Complete Atomic Boolean Algebra

A Complete Atomic Boolean Algebra B is given by the following axioms, adapted from Hughes and Cresswell, 1973:

(i) B contains at least 2 elements.

(ii) If $a, b \in B$ then $a' \in B$ and $a \sqcup b \in B$.
(iii) If $a, b \in B$ then $a \sqcup b = b \sqcup a$.
(iv) If $a, b, c \in B$ then $a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c$.
(v) For all $a, b \in B$, if there is some $c \in B$ such that $a \sqcup b' = c \sqcup c'$ then $a \sqcup b = a$.
(vi) For all $a, b, c \in B$ if $a \sqcup b = a$ then $a \sqcup b' = c \sqcup c'$.
(vii) For all $a \neq (c \sqcup c') \in B$ there exists $u \in B$, such that $u \sqcup a = q$ and for all $i \in B$ such that $i \sqcup u = u$ either $i = (c \sqcup c')$ or $i = u$.
(viii) Any (non-empty) set $E \subseteq A$ has a least upper bound, $\bigsqcup E \in A$.

The axioms (i)–(vi) give a Boolean algebra. Axiom (vii) makes the algebra atomic. Axiom (viii) makes it complete. The notions *bottom* **0**; *top* **1**; $a \sqcap b$; and $a - b$ can be defined:

$$\mathbf{0} =_{\text{def}} a \sqcup a'$$
$$\mathbf{1} =_{\text{def}} \mathbf{0}'$$
$$a \sqcap b =_{\text{def}} (a' \sqcup b')'$$
$$a - b =_{\text{def}} a \sqcap b'$$

There is one well-known theorem that will be of use later:

Lemma A.1 *In an atomic Boolean algebra, every element is the supremum of the atoms it dominates (each element of B is* defined *by the atoms it dominates).*

Proof: From Halmos, 1963: each element $p \in B$ is an upper-bound of the set of atoms $E \subseteq B$ that it dominates. We must demonstrate that if r is an arbitrary bound of E, then $p \sqsubseteq r$. Assume otherwise, that $p - r \sqsubseteq \mathbf{0}$. From atomicity it follows there is a $q \in B$ where $q \sqsubseteq p - r$. As $p - r \sqsubseteq p$, the atom $q \in E$. But since $(q \sqcap r) \sqsubseteq ((p - r) \sqcap r)$ this contradicts that r is an upper-bound of E. □

We can draw two corollaries from this result:

Corollary A.1 *Different collections of atoms have different suprema.*

Corollary A.2 *Different suprema dominate different atoms.*

Definition A.3 *Let $\mathcal{M}_\Delta = \langle \mathcal{D}, T, P, B \rangle$ be a model of* PT+Δ. *Where $B : D \longrightarrow \{0, 1\}$ characterises those elements of D that are in a complete atomic Boolean sub-domain of D, with the same ordering and join operator.*

We can now express the conditions for $\delta t, t \leq t'$:

$$\mathcal{M}_\Delta \models_g \mathrm{T}(\delta(t)) \quad \text{iff} \quad B(\mathcal{I}[t]_g) = 1$$
$$\mathcal{M}_\Delta \models_g t \leq t' \quad \text{iff} \quad \mathcal{I}[t]_g \sqsubseteq \mathcal{I}[t']_g$$

Thus, natural language denotable terms denote items in the complete atomic Boolean algebra.

As it does not matter what 'non-denoting' definite descriptors denote, the definite descriptor $\sigma x \varphi$ can be given an interpretation:

$$\mathcal{I}[\sigma x \varphi]_g \quad = \quad \bigsqcup \{a | T(\mathcal{I}[\varphi]_{g[a/x]}) = 1\}$$

It can be demonstrated that the model satisfies the axioms given for denotable terms.

Theorem A.1 *The summation operator ⊕ is symmetric, idempotent and associative (Axioms 3.1; 3.2; 3.3):*

Proof: Trivial: the summation operator is modelled by the lattice theoretic join ⊔ which is also symmetric, idempotent and associative. □

Theorem A.2 *The domain of denotables is closed (Axiom 3.5).*

$$\Delta a \,\&\, \Delta b \rightarrow \Delta(a \oplus b)$$

Proof: From the axioms of the Boolean algebra: if $a, b \in B$, then $a \sqcup b \in B$. □

Theorem A.3 *The domain of denotables is (definably) complete (Axiom 3.6).*

$$\forall p((\mathrm{Pty}(p) \,\&\, \forall x(\mathrm{T}(px) \rightarrow \Delta(x)) \,\&\, \exists x(\mathrm{T}(px))) \rightarrow \Delta(\sigma x p x))$$

Proof: From the completeness of the Boolean algebra: if $X \subseteq B$, then $\bigsqcup X \in B$. The axiom is weaker, as it requires that there is a denotable in the extension of the property. □

Theorem A.4 *Different denotables have different parts (Axiom 3.7).*

$$\forall_{\Delta} xy(x \not\leq y \rightarrow \exists_{\Delta} u(u \leq x \,\&\, u \not\leq y))$$

Proof: From Corollary A.2, we can prove that for all denotable x, y, if $x \not\leq y$ then there is a denotable u that is part of x but not part of y, and that denotable is atomic in the model. This supports Axiom 3.7, which has a weaker consequent: it does not require u to be atomic. □

Theorem A.5 *Denotables within the extension of a property (of denotables) must have an upper-bound (Axiom 3.8).*

$$\forall p \forall_{\Delta} y((\mathrm{Pty}_{\Delta}(p) \,\&\, \mathrm{T}(py)) \rightarrow y \leq \sigma_{\delta} x p x)$$

Proof: The axioms of the Boolean algebra define $\bigsqcup X$ as an upper-bound on the members of the set $X \subseteq B$. The supremum operator σ is modelled by $\bigsqcup$. Further, all PT definable properties of denotables will have their extension in B. □

Theorem A.6 *The supremum of the extension of a property of denotables is the smallest denotable that dominates all the terms in the extension (Axiom 3.9).*

$$\forall p \forall_{\Delta} y((\mathrm{Pty}_{\Delta}(p) \,\&\, \forall_{\Delta} x(\mathrm{T}(px) \rightarrow x \leq y)) \rightarrow \sigma_{\delta} x p x \leq y)$$

Proof: From the axioms of the Boolean algebra: $\bigsqcup X$ is the *least* upper-bound of X, where $X \subseteq B$. □

Theorem A.7 *Different portions of denotables have different suprema (Axiom 3.10).*

$$\forall p \forall_{\Delta} u((\mathrm{Pty}_{\Delta}(p) \,\&\, u \leq \sigma_{\delta} x p x) \rightarrow (\exists_{\Delta} z(pz \,\&\, u \leq z) \vee \exists_{\Delta} z(pz \,\&\, z \leq u)))$$

Proof: From Corollary A.1 we can prove that for all p, u, where p is a property of denotables and u is a denotable, if u is atomic in the model, and it is part of the supremum of the denotables that are p, then u is part of some denotable z in the extension of p. However, the theory is not atomic. If there is no mention of atomicity in the antecedent, then the model will support the axiom if the consequent is weakened to also allow u to be part of some term in the extension of p. □

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