Imperatives: a logic of satisfaction*

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Abstract

The paper discusses some issues concerning the semantic behaviour of imperatives, presents a proof-theoretic formalisation that captures their semantic behaviour in terms of propositional satisfaction criteria, and relates this approach to some existing proposals. It can be viewed as an attempt to formalise a "logic of satisfaction", as described by Hare (1967). Conditional imperatives and pseudo-imperatives are also considered, with some consideration of the nature of "practical inference" where propositions and imperatives appear together (Kenny, 1966). The issues that arise concerning disjunction introduction are discussed, including Ross' Paradox (Ross, 1941). A complementary notion of "refinement" for imperatives is introduced that captures "validity" relationships between commands. The theory is compared with some other proposals in the literature.

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1 Background

If we are interested in the formal semantics of natural language, then it is important to have an account of the meaning of expressions other than just indicatives. Utterances that don't immediately lend themselves to semantic analysis in some propositional language include questions, answers, non-sentential expressions and imperatives.

This paper concentrates on the latter. Imperatives are common in every day language, command systems, rules and instructions. Although there have been various proposals and sketches for the semantics of imperatives, there does not appear to be a generally agreed "first-approximation" to their basic formal semantic behaviour. The objective of the paper is to produce a formally weak, tractable theory that has *relevant* expressivity, and that is at an appropriate level of abstraction to avoid troublesome notions, such as causality.

There are many questions that can be asked about imperatives, concerning what they are and how they behave, including how they are best characterised syntactically, and how they relate to other pheneomena, such as deontic modals. There are also questions about expressions that are of imperative form, but are not usually interpreted as imperatives, and non-imperative expressions that are. Here we will initially focus on foundational issues in the semantics of expressions that, generally speaking, lie outside such gray areas.

To this end, we will start off by taking insights from examples of basic clear-cut imperatives, such as

(1) *"Shut the door!"*

building up to some more complex cases (conditional imperatives, pseudo-imperatives). We will ignore 'imperatives' that don't have 'normal' imperative interpretations

(2) *"[May you] Live long and prosper!"*

and other constructions that have imperative intent

(3) *"Could you pass the salt please?"*

The methodology taken follows the suggestion of Huntley (1984) that the objective of a semantic theory of imperatives should be to abstract away from different illocutionary uses of imperatives (Broadie, 1972; Hamblin, 1987), and to capture some relevant "core meaning", as is the accepted practice with the semantics of indicatives.[1]

One principle adopted here is to take an objective, neutral view of the meaning of an imperative where we are not interested in modelling an individual's personal response to an imperative or the state of mind of the person uttering an imperative (following Jørgensen (1937–38)). We also avoid issues concerning discourse and multi-agent contexts, much as is the case with a simple propositional logic for indicatives.

[1] This approach is also advocated by Lappin (1982).

1.1 Some Semantic Issues

There are a range of semantic issues that have to be considered when formulating a semantic theory of imperatives. These include: 'inference' patterns; felicity conditions; disjunction of imperatives; negation of imperatives; conditional imperatives; pseudo-imperatives; 'pragmatic' effects. Here we discuss some of these issues in turn.

1.1.1 Negation

Hamblin considered the various types of negation that imperatives appear to enjoy.[2] For reasons of simplicity we will concentrate on Hamblin's Type I and Type II negation. Given the sentence:

(4) *"Don't come to the party!"*

according to Hamblin we can interpret this to mean either

(5) Don't attend the party.

or

(6) Don't take steps to attend the party.

The intuition is perhaps clearer if we consider utterances such as

(7) *"Don't climb the mountain!"*

which could be taken to mean either don't ascend the mountain (all the way to the top), or don't climb on the mountain (anywhere).

It is worth observing that this just appears to be a form of telic/atelic ambiguity. If we can find an appropriate level of abstraction in our account of imperatives, we might hope to capture the ambiguity in the imperative by way of the same telic/atelic ambiguity that appears in indicative utterances.

In general, there is a wider issue that arises with negated imperatives. We have to consider how utterances of the form

(8) *"Don't buy an apple!"*

are to be interpreted. We can conceive of action-based formulations of the semantics of imperatives[3] where ascertaining whether an agent has complied with this imperative may generally require quantification over all the actions of an agent, and all the logical and causal consequences of those actions. Formally speaking, this is a version of what is known as the "frame problem" of artificial intelligence (McCarthy & Hayes, 1969). One aspect of this problem is providing the formal machinery to express the fact that most things remain the same in the face of an individual action. Perhaps a more troubling issue

[2] Vander Linden & Di Eugenio (1996) provide a corpus study of some examples of negation with imperatives.

[3] Often the precise role and nature of actions in a given theory is not entirely cleary, but some conception of an action that is independent of some propositional description is clearly assumed by Pérez-Ramírez & Fox (2003a), Pérez-Ramírez & Fox (2003b) and Franke (2005b) and the model of Lascarides & Asher (2004), for example.

here is whether or not we can determine that something is causally unaffected by other changes (in this case, that an apple remains unbought despite other actions of the agent).

One of the fundamental objectives of the formalisation given in this paper is to allow proof-theoretic characterisation of the behaviour of imperatives, including negated imperatives, which captures our core intuitions about their behaviour without becoming directly dependent on finding a solution to the complexities and problems of analysing cause-and-effect relationships.

1.1.2 Disjunction

Disjunctive imperatives appear to be ambiguous. In the following example, a disjunctive imperative is given in answer to a question[4], and the imperative answer supports two interpretations, which are clarified by the alternative continuations.

(9) (a) *"What do I have to do?"*

(b) *"Prepare the lecture or mark the exams...*

i. *...it's up to you."*

ii. *...we don't yet know which."*

The interpretation of the imperative which supports (i) is known as a free-choice, or choice-offering interpretation (Aloni, 2003; Kamp, 1973; Hamblin, 1987). This ambiguity also arise with deontic modals:

(10) *"You should prepare the lecture or mark the exams."*

In the case of deontic modality, the ambiguity could be represented as a scoping ambiguity of *"you should"* with respect to *"or"*, or perhaps more precisely, an ambiguity in whether or not we permit *"you should"* to distribute across *"or"*.

(11) (a) *"You should (prepare the lecture or mark the exams)."*

(b) *"You should prepare the lecture or [you should] mark the exams."*

This suggests that some kind of scoping/distribution ambiguity might be used to encode the two readings of disjunction that arise with imperatives.

Concerning disjunction and imperatives, there is also the issue of Ross' Paradox (Ross, 1941), which will be touched on in Section 1.1.4 and discussed in more detail in Section 3.2, which concerns the desirability, or otherwise, of inferences of the form

(12)
$$\frac{\text{"Post the letter!"}}{\text{"Post the letter or burn the letter!"}}$$

[4] As noted by Hamblin (1987), imperatives may often be given in answer to questions, although they need not always be interpreted as commands (Lascarides & Asher, 2004).

1.1.3 Felicity Conditions

There are circumstances when imperatives do not appear to be felicitous, because it is not possible to comply with them, either through circumstance or lack of ability.

(13) (a) *"Shut the door!"*

(b) i. *"But the door is already shut."*

ii. *"But I am chained to the chair."*

This is sometimes described as being about the *validity*, *satisfiability* or *correctness* of an imperative (van Eijck, 2000; Pérez-Ramírez & Fox, 2003a).[5] We could also consider cases where compliance with an imperative allows another imperative to be satisfied, or *entailed* (van Eijck, 2000).

One question concerns whether, and in what way, this issue should be addressed in a formal theory. We could consider the felicity conditions to be a constraint on well-formedness, so that the utterance is not even counted as an imperative if it is infelicitous. Alternatively we could incorporate the felicity conditions at some other point in the theory.

Rather than being prescriptive about the best way of tackling this issue in the formal theory, it might be worth just observing that this issue also arises with indicatives, in the form of presuppositions.

(14) (a) *"John shut the door."*
[Presupposes the door was open]

(b) *"Have you stopped beating your wife?"*
[Presupposes you have been beating your wife]

(c) *"The present king of France. . . "*
[Presupposes there is a king of France]

The question is then whether the felicity conditions of imperatives really require special attention, or just some adaption of a treatment of presuppositions.[6]

One argument for not ruling out infelicitous imperatives entirely comes when considering disjunctive imperatives (Section 2.5, page 19), such as

(15) *"Open the door or keep it open!"*

where the relevant response depends upon which disjunct can be satisfied.

1.1.4 Inference

Although imperatives are not usually considered to be expressions that have truth values — which renders them distinct from propositions — patterns of 'entailments' between imperatives can be observed (Jørgensen, 1937–38; Ross, 1941; von Wright, 1963b; Hare, 1967; Chellas, 1971; Segerberg, 1990). For example, from

[5] Not to be confused with the notion of validity employed by Ross (1941) and others, meaning inferences that are consistent with the wishes of the person uttering the imperative (Section 3.2).

[6] Note that the presuppositions of definite descriptors arises in both cases; *"But there is no door!"*

(16) *"Shut the door and close the window!"*

we can infer we are being requested to

(17) *"Shut the door!"*

This is akin to the conjunction elimination rule of propositional logic, and suggests that imperatives do support proposition-like behaviour, perhaps due to some proposition-like content, or by way of a relationship to propositional expressions (described as "the standard approach" by Huntley (1984)), or to expressions of some more fundamental algebraic category that underpins a range of utterance types, including imperatives and indicatives Hare (1949).[7]

In the connection with this inferential behaviour, one issue that has received much comment is Ross' 'paradox', alluded to above in Section 1.1.2. This goes roughly as follows. From

(18) *"Shut the door!"*

we might 'infer'

(19) *"Shut the door or close the window!"*

However, satisfying the consequent by closing the window does not mean we have satisfied the antecedent. Furthermore, it would be wrong to appeal to the authority of the antecedent in order to justify an action of closing the window, which satisfies one of the antecedent's 'logical consequences'. We will revisit this issue later, but will just observe here that many of the problematic issues for imperatives also arise with other kinds of sentences, such as indicatives and modal statements.

In this paper, we seek to capture the apparent entailment behaviour of imperatives by use of proof rules over imperatives and their satisfaction criteria, whilst allowing imperatives to be of a distinct type which is not to be equated with propositions.

Later (Section 3.1, we will also touch upon the issue of "practical inference" (Kenny, 1966), where imperatives and propositions appear together within a deduction, as in

(20)
$$\frac{\textit{"If someone is ill, give them an injection"} \quad \textit{"John is ill"}}{\textit{"Give John an injection"}}$$

Arguably, the desire to handle such inferences is one reason why reductions from imperatives to propositions have seemed so attractive, and why disjunction introduction — as in example (12) — has seemed so unappealing.

1.1.5 Combining Propositions and Imperatives

In addition to the above "practical inferences", there appear to be more overt ways in which indicatives and imperatives may be combined, as in the case of conditional imperatives (Section 2.7) and pseudo-imperatives (Section 2.8

[7] See Section 4.

and 2.9. Arguably, the results of such combinations may be either imperative or propositional, or even both (in the case of pseudo-imperatives, see Franke (2005b) for example). We require a theory that has a type system that is sufficiently flexible to accommodate such intuitions.

1.2 The General Approach

The formal theory of this paper seeks to capture the formal 'logical' behaviour of imperatives by way of inference rules over their propositional *satisfaction criteria*. In this paper, these criteria are taken to be propositional descriptions of the relevant actions *by the relevant agent*. For example, the imperative

(21) *"Shut the door!"*

addressed to an agent John would be satisfied by any action that is felicitously described by the propositional content of

(22) *"John shuts the door."*

in the salient future. *And that we can then consider relationships between imperatives by way of the relationships between their satisfaction criteria.*

Unlike other proposals, we do not consider the relevant action *directly* within the formalisation, nor do we take the action to be something that is specified by a non-agentive extensional outcome, such as the state in which *"The door is shut."* This side-steps problems with causation and related difficulties that can arise if we have actions, as such, in the semantic account.

One useful consequence of this approach is that any Type I and Type II negation ambiguity (Section 1.1.1) can be captured by an exactly parallel telic/atelic negation ambiguity in the satisfication criteria. It should also be possible to express felicity conditions for imperatives, analysed in terms of the presuppositional content of the propositions which express the satisfaction criteria (see Section 4.1).

The motivation behind the notion of "satisfaction criteria" is similar to that of "fulfilment criteria" and "outcomes" proposed by Lappin (1982) and Ginzburg & Sag (2000), respectively, as a way of capturing the meaning of imperatives. In particular, it is in the spirit of the proposed "logic of satisfaction" or fulfilment outlined by Hare (1967).[8] Here, however, the objective is to capture the inferential patterns of imperatives and satisfaction criteria directly, largely free of any specific philosophical and notational commitments.

In formulating the proof rules concerning imperatives and their satisfaction criteria, the general methodology is adopted of using a weak, tractable theory that has *relevant* expressivity at an appropriate level of abstraction[9] and which avoids troublesome notions, such as theories of action and causality.

[8] Hare credits Ross (1941) and Kenny (1966) for drawing something like his distinction between the logic of validity for indicatives and the logic of satisfaction for imperatives. The idea of a logic of satisfaction is also raised in earlier work, such as Hofstadter & McKinsey (1939) (Section 6.3).

[9] This is in keeping with the principles described in Fox (2000).

The style of inference system adopted includes typing rules that, in effect, express well-formedness criteria for imperatives, propositions, and their combination, and which allow us to determine to which type an expression belongs (if any).

In addition to these categorial typing rules, there are proof rules that serve to give:

(a) the truth conditions of complex propositions;

(b) the satisfaction conditions for complex imperatives;

(c) and whatever is relevant for hybrid expressions (typically a combination of truth conditions and satisfaction conditions).

Once we have established a collection of rules which *prima facia* capture our intuitions about the expressions in questions we should then proceed to find a model for these rules in order to demonstrate that they are able to support a consistent interpretation, i.e., to prove that the rules are not inconsistent. In this paper, we provide a sketch of such a model (Appendix A).

2 Formal Theory

First we sketch some salient aspects of a proof-theoretic treatment of a logic of satisfaction for imperatives. The essential idea is that from an agentive property p we can derive a proposition $p(\alpha)$ or an imperative $p!_\alpha$ where α is the understood agent, the person to whom an imperative is directed in the case of $p!_\alpha$. We can then write $\blacklozenge\, p!_\alpha$ to denote the satisfaction conditions of $p!_\alpha$. This will be a proposition that is true if the imperative is satisfied. We can then devise proof rules involving the satisfaction conditions of imperatives. By adopting this level of abstraction, we do not need to appeal to actions and events or causation.

The focus of this work is not so much to give a definitive account of imperatives or their satisfaction, rather, the main issue that we are concerned with here is demonstrating that such a proof-theoretic logic is possible, and makes sense. Unlike a similar proposal by Hofstadter & McKinsey (1939), we can see that considering full imperatives rather than a constrained set of fiats gives rise to a non-trivial extension to propositional logic.

The rest of the paper will then be devoted to justifying this approach in the face of well-known criticisms from Ross (1941) and others. Readers who do not wish to be distracted by the formal details of this theory may want to skip directly to Section 3.

In general, imperatives appear to be based upon agentive notions. Informally, we will have a class of agentive properties that, conceptually at least, can be considered to be agentive propositions with an abstracted subject, such as

(23) *"$\lambda x.x$ closes the door"*

We take such agentive properties to be basic.[10] If p is an agentive property, we will write $p(\alpha)$ to denote the property ascribed to some agent α, for example

(24) *"John closes the door."*

And $p!_\alpha$ will be used to denote the expression converted into an appropriate imperative form, addressed to an agent α, such as

(25) *"close the door[, John]!"*

The idea of imperatives being founded on agentive properties of agentives echos proposals by Castañeda (1975) and Anderson (1962). In the former case, the "propositional content" of imperatives is characterised as being sentences of the form *"α is to $\langle verb \rangle$"* (Castañeda, 1975, page 169). In the latter case, the propositional content is of the form *"α sees to it that Q"* (Anderson, 1962). Here we do not equate an imperative with such propositional content, although there is a similar propositional content implicit in their satisfaction criteria.

Note that there are many difficult issues concerning the precise nature of the sentences represented by $p(\alpha)$ and $p!_\alpha$, and the agentive property p, and the various relationships between them.[11] We are putting these details to one side in order to simplify the presentation of this account of their semantics. The objective is to be able to see that in some sense the proposition $p(\alpha)$, if true in some salient future, can be taken to 'satisfy' the directed imperative $p!_\alpha$. If we use $\diamond$ to denote a future tense modal operator, then, in the case of simple atomic imperatives at least, we can say that they are satisfied when $\diamond p(\alpha)$ is true.

Rather than equating the satisfaction of imperatives directly with such future tense judgements in all cases, we shall instead write $\blacklozenge i$ to denote the claim that an imperative i is satisfied. This makes it easier to consider the satisfaction relationships between more complex expressions which might not naturally be captured by any common theory governing the tense operator $\diamond$.

The formal theory itself is presented in terms of rules that govern the typing of expressions, the truth conditions for propositions, and the satisfaction conditions for imperatives. The typing rules allow us to decide whether a given expression represents a proposition, an imperative, a directed imperative, or whether it is considered to be a hybrid expression that is a combination of these types. The truth rules allow us to determine how the truth of a proposition depends upon the truth of its parts. The satisfaction rules allow us to show how the satisfaction of a complex imperative relies on the satisifaction of its parts. Hybrid expressions may have both truth conditions and satisfaction conditions.

The way the typing of the theory is set up allows us to formulate constraints on the semantic combinations of imperatives with propositions, as with conditional imperatives (Section 2.7), and pseudo imperatives (Section 2.8

[10] It is possible that basing the theory on agentive properties might help also account for modal complements with deontic expressions.

[11] For example, it could be argued that the p in $p!_\alpha$ corresponds to the interpretation of imperatives as denoting properties of agents (Hausser, 1978, 1980; Portner, 2005).

and 2.9). Essentially, complex imperative clauses can be built up using appropriate operations between constituent agentive properties, but these clauses can only be combined with propositional expressions after first being turned into a "completed" imperative.

2.1 Typing Rules

We simplify things by restricting ourselves to agentive properties ($\mathsf{Pty}_{\mathsf{ag}}$), imperatives that are directed towards some agent (α), and propositions (Prop) that include ascriptions of properties to the (fixed) agent. In addition to the types, we also have a truth judgement.

(26) Basic Types
$\mathsf{Pty}_{\mathsf{ag}}$ — *Agentive properties*
Imp — *Imperatives (uttered & directed)*
Prop — *Propositions*
True — *Truth judgement (of a proposition)*

Now that we have these basic types, we can start to express rules of inference that allow us to infer the types of more complex expressions.[12]

(27) Atomic Typing Rules

$$\text{(a)}\ \frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{p!_\alpha : \mathsf{Imp}}\, !_\alpha\mathcal{F} \qquad \text{(b)}\ \frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{p(\alpha) : \mathsf{Prop}}\, (\alpha)\mathcal{F}$$

We can define a number of typing rules for operators over both propositions and properties. In the case of imperatives, the typing rules will be introduced as we go along.

One rule to which we will appeal implicitly is that satisfaction conditions are propositions.

(28) Satisfaction conditions are propositions

$$\frac{i : \mathsf{Imp}}{\blacklozenge i : \mathsf{Prop}}\, \blacklozenge\mathcal{F}$$

Here it is assumed that there are appropriate independent logical connectives for propositions as well as properties. An alternative would be to express everything in terms of connectives between properties. This might have been more in keeping with the proposal of Hare (1949), but this has been avoided for reasons of clarity.

(29) Proposition operators

$$\text{(a)}\ \frac{\phi, \psi : \mathsf{Prop}}{\phi \wedge \psi : \mathsf{Prop}}\, \wedge\mathcal{F} \qquad \text{(b)}\ \frac{\phi, \psi : \mathsf{Prop}}{\phi \vee \psi : \mathsf{Prop}}\, \vee\mathcal{F}$$

[12] Here we will use $\mathcal{F}$ to denote rules of formation, $\mathcal{I}$ for introduction rules, and $\mathcal{E}$ for elimination rules. In general $p, q, \ldots$ will be used for agentive properties, $\phi, \psi, \ldots$ for propositions and $i, j, \ldots$ for imperatives, although formally there is no significance in the choice of these variable names.

$$\text{(c)}\ \frac{\phi, \psi : \mathsf{Prop}}{\phi \to \psi : \mathsf{Prop}} \to \mathcal{F} \quad \text{(d)}\ \frac{\phi : \mathsf{Prop}}{\neg\, \phi : \mathsf{Prop}} \neg\, \mathcal{F}$$

(30) Property operators

$$\text{(a)}\ \frac{p, q : \mathsf{Pty}_{\mathsf{ag}}}{p \cap q : \mathsf{Pty}_{\mathsf{ag}}} \cap \mathcal{F} \quad \text{(b)}\ \frac{p, q : \mathsf{Pty}_{\mathsf{ag}}}{p \cup q : \mathsf{Pty}_{\mathsf{ag}}} \cup \mathcal{F} \quad \text{(c)}\ \frac{p : \mathsf{Pty}_{\mathsf{ag}}}{-\, \phi : \mathsf{Pty}_{\mathsf{ag}}} - \mathcal{F}$$

There may be a connection between these (agentive) properties and Hare's notion of a neutral descriptive content that can be transformed into either an imperative or an indicative by way of some "dictive" function (Hare, 1949). This connection is not explored here, nor is any connection with grammatical imperative mood marking that arises in some languages (e.g. Greek and Hebrew).[13]

We leave the nature of an agent unanalysed. Typically an agent is taken to be some individual, but it might be appropriate to permit collections of individuals, corporate entities and machines to count as agents. Further it might be appropriate for an agent to delegate to another agent, or some part of itself. These issues are no doubt of relevance in a more detailed analysis of imperatives and other semantic phenomena. Here we leave such issues unanalysed in order to concentrate on the core patterns of behaviours of imperatives, rather than becoming too distracted by more general issues that impinge upon a broad range of semantic phenomena.

In general, imperatives are satisfied by actions that take place in some salient future.[14] As such, the propositional descriptions of these actions are usually in the future tense, for which we shall use the modal operator F.

(31) Future tense: we shall write $\diamond p$ to mean that p is true in the some (salient) future period.

$$\frac{\phi : \mathsf{Prop}}{\diamond \phi : \mathsf{Prop}} \diamond \mathcal{F}$$

2.2 Truth Judgements

Before moving on to consider imperatives and their satisfaction conditions, we need to complete the presentation of the basic theory of propositions and properties by giving the relevant rules governing truth judgements.

(32) Truth judgements for propositional conjunction

$$\text{(a)}\ \frac{\phi, \psi : \mathsf{Prop} \quad \phi\ \mathsf{True} \quad \psi\ \mathsf{True}}{\phi \wedge \psi\ \mathsf{True}} \wedge \mathcal{I}$$

$$\text{(b)}\ \frac{\phi, \psi : \mathsf{Prop} \quad \phi \wedge \psi\ \mathsf{True}}{\phi\ \mathsf{True}} \wedge \mathcal{E}_l \quad \text{(c)}\ \frac{\phi, \psi : \mathsf{Prop} \quad \phi \wedge \psi\ \mathsf{True}}{\psi\ \mathsf{True}} \wedge \mathcal{E}_r$$

[13] As has often been observed, grammatical categories are not always a reliable guide to the appropriate semantic analysis.

[14] It has however been argued by some (Rosja Mastop, Wim van der Wurff and others) that past tense imperatives do occur in some contexts with some languages (Mastop, 2005). See also Wolf (2007).

(33) Truth judgements for propositional disjunction

(a) $$\frac{\phi, \psi : \mathsf{Prop} \quad \phi \ \mathsf{True}}{\phi \vee \psi \ \mathsf{True}} \vee \mathcal{I}_l$$

(b) $$\frac{\phi, \psi : \mathsf{Prop} \quad \psi \ \mathsf{True}}{\phi \vee \psi \ \mathsf{True}} \vee \mathcal{I}_r$$

(c) $$\frac{\phi, \psi : \mathsf{Prop} \quad \begin{array}{c}[\phi \ \mathsf{True}] \\ \vdots \\ \chi \ \mathsf{True}\end{array} \quad \begin{array}{c}[\psi \ \mathsf{True}] \\ \vdots \\ \chi \ \mathsf{True}\end{array} \quad (\phi \vee \psi) \ \mathsf{True}}{\chi \ \mathsf{True}} \vee \mathcal{E}$$

(34) Truth judgements for propositional implication

(a) $$\frac{\phi, \psi : \mathsf{Prop} \quad \begin{array}{c}[\phi \ \mathsf{True}] \\ \vdots \\ \psi \ \mathsf{True}\end{array}}{\phi \to \psi \ \mathsf{True}} \to \mathcal{I}$$

(b) $$\frac{\phi, \psi : \mathsf{Prop} \quad \phi \to \psi \ \mathsf{True} \quad \phi \ \mathsf{True}}{\psi \ \mathsf{True}} \to \mathcal{E}$$

(35) Truth judgements for propositional negation

(a) $$\frac{\phi : \mathsf{Prop} \quad \begin{array}{c}[\phi \ \mathsf{True}] \\ \vdots \\ \bot \ \mathsf{True}\end{array}}{\neg \phi \ \mathsf{True}} \neg \mathcal{I}$$

(b) $$\frac{\phi : \mathsf{Prop} \quad \begin{array}{c}[\neg \phi \ \mathsf{True}] \\ \vdots \\ \bot \ \mathsf{True}\end{array}}{\phi \ \mathsf{True}} \neg \mathcal{E}$$

where $\bot$ is any contradictory statement of the form $(\psi \wedge \neg \psi) \ \mathsf{True}$.

(36) Truth judgements for property conjunction

(a) $$\frac{p, q : \mathsf{Pty}_{\mathsf{ag}} \quad p(\alpha) \wedge q(\alpha) \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{(p \cap q)(\alpha) \ \mathsf{True}} \cap \mathcal{I}$$

(b) $$\frac{p, q : \mathsf{Pty}_{\mathsf{ag}} \quad (p \cap q)(\alpha) \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{p(\alpha) \wedge q(\alpha) \ \mathsf{True}} \cap \mathcal{E}$$

(37) Truth judgements for property disjunction

(a) $$\frac{p, q : \mathsf{Pty}_{\mathsf{ag}} \quad p(\alpha) \vee q(\alpha) \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{(p \cup q)(\alpha) \ \mathsf{True}} \cup \mathcal{I}$$

(b) $$\frac{p, q : \mathsf{Pty}_{\mathsf{ag}} \quad (p \cup q)(\alpha) \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{p(\alpha) \vee q(\alpha) \ \mathsf{True}} \cup \mathcal{E}$$

(38) Truth judgements for property negation

(a) $$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \neg\, p(\alpha)\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{-\, p(\alpha)\ \ \mathsf{True}} -\mathcal{I}$$

(b) $$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad -\, p(\alpha)\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\neg\, p(\alpha)\ \ \mathsf{True}} -\mathcal{E}$$

(39) Future tense:

(a) $$\frac{\phi : \mathsf{Prop} \qquad \begin{array}{c}\\ \vdots \\ \phi\ \ \mathsf{True}\end{array}}{\Diamond\phi\ \ \mathsf{True}}$$

(b) $$\frac{\phi, \psi : \mathsf{Prop} \quad \Diamond(\phi \wedge \psi)\ \ \mathsf{True}}{\Diamond\phi \wedge \Diamond\psi\ \ \mathsf{True}}$$

(c) $$\frac{\phi, \psi : \mathsf{Prop} \quad \Diamond(\phi \vee \psi)\ \ \mathsf{True}}{\Diamond\phi \vee \Diamond\psi\ \ \mathsf{True}}$$

(d) $$\frac{\phi, \psi : \mathsf{Prop} \quad \Diamond\phi \vee \Diamond\psi\ \ \mathsf{True}}{\Diamond(\phi \vee \psi)\ \ \mathsf{True}}$$

(e) $$\frac{\phi, \psi : \mathsf{Prop} \quad \Box(\phi \rightarrow \psi)\ \ \mathsf{True}}{\Box\phi \rightarrow \Box\psi\ \ \mathsf{True}}$$

(f) $$\frac{\phi : \mathsf{Prop} \quad \neg \Diamond\phi\ \ \mathsf{True}}{\Diamond \neg\, \phi\ \ \mathsf{True}}$$

where $\Box\phi$ is $\neg \Diamond \neg\, \phi$.

In fact, any reasonable propositional theory of future tense will suffice here (see Prior (1967) and van Benthem (1983), for example).

2.3 Atomic Satisfaction

We are now in a position to present rules governing satisfaction criteria and truth conditions, starting with the basic case. Given that the expressions are of an appropriate type, then a directed imperative $p!_\alpha$ is satisfied by an action which can be described by the proposition $\Diamond p(\alpha)$.

(40) Satisfaction (Atomic)

$$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \Diamond\, p(\alpha)\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge\, p!_\alpha\ \ \mathsf{True}}\ \blacklozenge\ \mathsf{True}_{\mathrm{atomic}}$$

This is intended to capture the following kind of satisfaction relationships.

(41) If it is true that *"(In the future) John closes the door"*, then *"Close the door [John]!"* is satisfied.

We might be tempted to say one of the following.

(42)

$$\text{(a)}\ \frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge\, p!_\alpha\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\diamond p(\alpha)\ \ \mathsf{True}} \qquad \text{(b)}\ \frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{\blacklozenge\, p!_\alpha = \diamond p(\alpha)}$$

Such a reduction may give the appropriate behaviour in some cases, but unfortunately not in all. For example, the proposed analysis for conjunction departs from the usual behaviour of conjunction in the future tense. We also need the option of considering satisfaction of imperatives that don't naturally have a direct propositional equivalent, as in the case of conditional imperatives and pseudo imperatives.

If we are dealing with atomic imperatives, then we can give a rule along the lines of (42) that allows us, in effect, to eliminate the satisfaction operator. We achieve this by introducing a type for atomic-agentive-properties $\mathsf{Pty}^{\mathsf{at}}_{\mathsf{ag}}$; agentive properties that cannot be decomposed. We can then also constrain the application of (40) to such properties. Atomic agentive properties can then be governed by the following rules.

(43) Atomic agentive properties

$$\text{(a)}\ \frac{p : \mathsf{Pty}^{\mathsf{at}}_{\mathsf{ag}}}{p : \mathsf{Pty}_{\mathsf{ag}}}$$

$$\text{(b)}\ \frac{p : \mathsf{Pty}^{\mathsf{at}}_{\mathsf{ag}} \quad \diamond\, p(\alpha)\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge\, p!_\alpha\ \ \mathsf{True}}\ \blacklozenge\mathcal{I}$$

$$\text{(c)}\ \frac{p : \mathsf{Pty}^{\mathsf{at}}_{\mathsf{ag}} \quad \blacklozenge\, p!_\alpha\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\diamond p(\alpha)\ \ \mathsf{True}}\ \blacklozenge\mathcal{E}$$

Now we can consider the typing rules and satisfaction criteria for more complex expressions.

2.4 Conjunction

We can combine imperatives by way of conjunction to form a complex 'conjoined' imperative.

(44) *"Shut the window and close the door!"*

In the semantics we could represent conjunction by way of $\cap$, regardless of what kinds of expressions are being conjoined. It is then up to the typing rules to determine whether any particular use of $\cap$ in a given context gives rise to a well-formed conjunction.

Satisfaction of such an imperative requires satisfaction of each of the conjuncts, and if such an imperative is satisfied, then we know that each conjunct has been satisfied. More formally, if $p!_\alpha$ is satisifed and $q!_\alpha$ is satisfied, then $(p \cap q)!_\alpha$ is satisfied. Furthermore, if $(p \cap q)!_\alpha$ is satisfied then $p!_\alpha$ and $q!_\alpha$ are also satisfied.

We are not claiming that *desiring* the satisfaction of $(p \cap q)!_\alpha$ means an agent desires the satisfaction of $p!_\alpha$ or $q!_\alpha$ individually; this is a logic of satisfaction not of desire (nor of *validity*, in the sense of Ross (1941)).

We can now give the formal rules that govern satisfaction.

(45) Typing

$$\frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{(p \cap p')!_\alpha : \mathsf{Imp}} \, ! \cap \mathcal{I} \text{ (a theorem)}$$

(46) Satisfaction of conjunctive imperatives

(a) $$\frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge p!_\alpha \ \mathsf{True} \quad \blacklozenge p'!_\alpha \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge (p \cap p')!_\alpha \ \mathsf{True}} \, ! \cap +$$

(b) $$\frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge (p \cap p')!_\alpha \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge p!_\alpha \ \mathsf{True}} \, ! \cap -_l$$

(b) $$\frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge (p \cap p')!_\alpha \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge p'!_\alpha \ \mathsf{True}} \, ! \cap -_r$$

As can be seen, satisfaction $\blacklozenge$ is a weaker notion than the future tense operator $\diamond$: unlike that tense operator, a conjunctive imperative is satisfied if each of the conjucts is satisfied, even if the actual satisfication is achieved at different points in the future. It may be possible to give constraints on agentive properties p or agentive propositions $p(\alpha)$ that achieve the same effect, but we leave these alternatives for others to pursue.

One issue not considered by these rules is that of temporal ordering, as is evident in the intended interpretation of

(47) *"Put on a parachute and jump out."* (Hare, 1967)

We will merely observe that such an account of temporal ordering, or narrative sequencing, is something required by indicative statements.

(48) *"John put on a parachute and jumped out."*

For this reason it is not considered a critical part of the analysis of core imperatives.[15]

2.5 Disjunction

In the case of disjunction, we could follow a similar approach to the treatment of conjunction, except that we have to deal with the ambiguity between conventional and free-choice interpretations of disjunction (Section 1.1.2). To this end, we shall allow disjunction ($\vee$) over imperatives as well imperatives formed from the disjunction ($\cup$) of constitutent agentive properties. The level

[15]These and similar issues are discussed by Lascarides & Asher (2004), Asher & Lascarides (2003) and Culicover & Jackendoff (1997).

at which the disjunction takes place can be used to capture whether we have Free Choice or Non-Free Choice disjunction (weak disjunction).

We can use $(p \cup q)!_\alpha$ to represent a Free Choice conjunction (α is commanded to satisfy either $p!_\alpha$ or $q!_\alpha$). Such a disjunction will be satisifed when either $p!_\alpha$ or $q!_\alpha$ is satisfied.

Non-Free Choice disjunction can be represented by $(p!_\alpha \vee q!_\alpha)$, meaning that α is commanded to ensure $p!_\alpha$, or α is commanded to do $q!_\alpha$. Ultimately, it is satisfied by the satisfaction of $p!_\alpha$ or by the satisfaction of $q!_\alpha$, although it is not clear which is required at the time of the utterance. Of course we may question how such a disjunction in itself can have the force of an imperative as opposed to acting as a guide or constraint on some other utterance or obligation which itself carries the full imperative force. The view we take here is that such an imperative is 'underspecified' in some sense. We capture this by considering which expressions do not satisfy such an imperative, rather than attempting to indicate directly which expressions will satisfy the imperative.

The distinction between free choice and weak disjunction is, arguably, somewhat akin to the notions of internal (non-deterministic) and external (deterministic) choice in process algebra (Hoare, 1978).[16] There is perhaps a conceptual difference however, in that choice in process algebra is considered in the context of defining the behaviour of a process that will respond to external events, whereas as we are concerned with disjunction as it appears to be present in external events themselves; namely the utterances of other agents.

We can now give the relevant formal rules.

(49) Typing

$$\text{(a)}\ \frac{p, p' : \mathsf{Pty}_{\mathsf{ag}}}{(p \cup p')!_\alpha : \mathsf{Imp}}\ !\cup\mathcal{I}\ \text{(a theorem)} \quad \text{(b)}\ \frac{i, i' : \mathsf{Imp}}{i \vee i' : \mathsf{Imp}}\ !\vee\mathcal{I}$$

We may question whether it is appropriate to consider $i \vee i'$ to be a constraint rather than an imperative as such.[17] For simplicity, we shall make a working assumption that it can be imperative in nature, without making any commitment to an analysis of the performative status of such constructions.

We start by giving what is hoped are fairly uncontroversial rules for Free Choice disjunction.

(50) Satisfaction (Free Choice)

$$\text{(a)}\ \frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge p!_\alpha\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge(p \cup p')!_\alpha\ \ \mathsf{True}}\ !\cup\mathcal{I}_l$$

[16] In a process algebra such as CSP (Hoare, 1978), deterministic choice $(a \to P)\Box(b \to Q)$ says that the process can respond to either a or b at this point (and then behave as P or Q respectively). The environment resolves the choice as the behaviour depends upon which (external) event, a or b, occurs at the relevant point. This seems somewhat similar to weak disjunction. Non-deterministic choice $(a \to P) \sqcap (b \to Q)$ says that the process itself decides whether or not it will respond to external event a or to event b; the environment has no control over which choice is made by the process (and hence to which event, a or b, the process will respond). This has some similarity to Free Choice disjunction.

[17] Ruth Kempson (PC).

$$\text{(b)}\ \frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge p'!_\alpha\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge(p \cup p')!_\alpha\ \ \mathsf{True}}\ ! \cup \mathcal{I}_r$$

$$\text{(c)}\ \frac{p, p' : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge(p \cup p')!_\alpha\ \ \mathsf{True} \quad \alpha : \mathsf{Agent}}{\blacklozenge p!_\alpha \vee \blacklozenge p'!_\alpha\ \ \mathsf{True}}\ ! \cup \mathcal{E}$$

This last rule is an elimination rule in the sense that it is eliminating disjunction between $\mathsf{Imp}_{\mathsf{Abs}}$ and replacing it by disjunction between Prop.

The Free Choice satisfaction rules effectively state that satisfying the command

(51) *"Go to the beach!"*

also satisfies

(52) *"Go to the beach or go to the cinema!"*

under a free choice interpretation, but the inference does not work in the other direction. In particular, it is not possible to infer that going to the cinema indirectly satisfies the command to go to the beach, the so-called Ross' Paradox (see Section 3).

The following rules deal with non-Free Choice introduction and elimination. They are expressed in terms of the failure of a proposition to express an appropriate satisfaction criteria. This works around the apparent underspecified nature of such disjunction.

(53) Satisfaction (Non Free Choice)

$$\text{(a)}\ \frac{i, i' : \mathsf{Imp} \quad \neg\blacklozenge i\ \ \mathsf{True} \quad \neg\blacklozenge i'\ \ \mathsf{True}}{\neg\blacklozenge(i \vee i')\ \ \mathsf{True}}\ ! \vee \mathcal{I}\ \text{(weak)}$$

$$\text{(b)}\ \frac{i, i' : \mathsf{Imp} \quad \neg\blacklozenge(i \vee i')\ \ \mathsf{True}}{\neg\blacklozenge i}\ ! \vee \mathcal{E}_l\ \text{(weak)}$$

$$\text{(c)}\ \frac{i, i' : \mathsf{Imp} \quad \neg\blacklozenge(i \vee i')\ \ \mathsf{True}}{\neg\blacklozenge i'}\ ! \vee \mathcal{E}_r\ \text{(weak)}$$

The weakness of the non-Free Choice disjunction should be evident from the proof rules: we are only able to say what does *not* satisfy the disjunction. Note that we can derive an equivalence between $\neg\blacklozenge i \wedge \neg\blacklozenge i'$ and $\neg\blacklozenge(i \vee i')$, which has the appearance of a De Morgan's law for $\neg\blacklozenge$.

The use of scoping to differentiate the free choice disjunction from the non-free choice disjunction is similar to Kamp's proposed F scoping operator, were $F(p) \vee F(q)$ and $F(p \vee q)$ were the Free Choice (strong) and non-Free Choice (weak) readings of disjunction, respectively (Kamp, 1973). In the case of the current theory, the relevant scoping appears to be the other

way around, with the Free Choice disjunction having narrower scope than the weaker disjunction[18]

These rules just consider the propositional and imperative level disjunction and Free Choice, and not any Free Choice (and non-Free Choice) ambiguity that may arise with some form of existential quantification, for example

(54) (a) *"Pick up an apple! . . . Any will do."*
(b) *"Pick up an apple! . . . I will tell you which one."*

or the even more ambiguous

(55) (a) *"Pick up a newspaper! . . . Any will do."*
(b) *"Pick up a newspaper! . . . I will tell you which one."*

as this lies outside the scope of the current formalism. Of course such issues need to be taken into account when extending the theory to include quantification.

It is conceivable that Free Choice might be offered to a collection of individuals, or some corporate entity, but with constraints upon who can engage in satisfying any given disjunct. This could be a variant of some of the examples considered by Hamblin (1987) where the imperative is directed at those who are expected to ensure that others bring about the desired result. In this case, the "others" in question would be members of the collection of individuals. For example, *"Find some fruit or catch some fish!"*, directed at a group of people might be expected to result in a collective or autocratic decision that some members of the group should engage in one or other activity. We shall not overtly consider such cases here, as this raises more general, thorny issues about agency in the context of delegation. Instead, we shall leave the notion of agency unanalysed.

We have not here considered cases of disjunction (Free and otherwise) where one of the disjuncts is not achievable, or where the felicity conditions are mutually exclusive. Examples would include the somewhat stilted

(56) *"Open the door or keep it open!"*

where only one disjunct may be fulfilled von Wright (1963b, page 159). The analysis of such examples should fit in with the analysis of conditional imperatives (Section 2.7), so that they have the same satisfaction behaviour as

(57) *"If the door is closed, open it, if it is open, keep it open!"*

Such examples provide an argument for not setting up the type system in such a way that it rules out infelicitous constituent imperatives (Section 1.1.3).[19]

[18] Kamp (1973) took the view that the Free-Choice reading was handled by the usual union-like interpreted of disjunction over possible-worlds, but that the weaker reading required some elaboration to maintain the union-like character of disjunction. Later, Kamp (1979) revised this view to one where the weaker reading was taken to be the natural one, with the Free-Choice reading required elaboration through rules of conversation or similar. The scoping chosen in the current analysis need not by itself undermine any claim that disjunction should universally be given a union-like interpretation.

[19] In a related point, von Wright also mentions the issue that different obligations may have different applicability conditions, which brings into question whether there can be straightforward entailment conditions between them (von Wright, 1963b, page 181).

It is worth noting that the rules given here allow the possibility of satisfying a Free Choice disjunctive imperative even if one of the disjuncts is not satisfiable. Given the information that one of the disjuncts is not satisfiable, it is possible to determine that the only way of satisfying a free disjunction is to satisfy the other disjunct.[20] Of course, it would be appropriate to formulate a notion of satisfiability (Section 4.1) to aid such reasoning. Even without this, arguments of the form

(58) *"Do a or b"*, *"do not do a"* therefore *"do b"*

are supported.[21]

2.6 Negation

At first sight, negated imperatives may appear to present some difficulties, as has been outlined above in Section 1.1.1. First there is the general issue of what is being required of an addressee when they are asked not to do something, and how it can best be represented. Second, as noted by Hamblin (1987), there appear to be different types of interpretations of negated imperatives.

Concerning the first point, an addressee is presumably being required to refrain from a particular action. This might present difficulties for theories that seek to interpret imperatives as specifying explicit actions. We would then have to quantify over all the possible future actions of the addressee and assert that the prohibited action is not among those actions, and does not follow from them logically or causally. The level of abstraction at which we are working helps us avoid these difficulties. In the current approach, the negated imperative is satisfied if the addressee can simply be *described* as having refrained from the relevant activity in the salient future. The imperative

(59) *"Don't close the door!"*

can be represented by $-p!_{\alpha}$. This is then satisfied if $-p(\alpha)$ is true (i.e. if $\neg\, p(\alpha)$ is true) corresponding to the sentence

(60) *"α does not close the door (ever)."*

This approach generally avoids the problem of characterising negative actions (avoiding/preventing something from coming about).

[20] Although perhaps one role of imperatives, particularly free choice imperatives, is to convey the fact that certain possibilities are available to the addressee — a view attributed to Belnap, Lewis and others (Hans Kamp, PC). A formal analysis of this kind of meaning would presumably be based upon the presuppositions of the imperatives, and some notion of accommodation (see Stalnaker (1973) for example). Such an analysis is beyond the scope of the current paper.

[21] Williams (1963) says such arguments should not be permitted as there is a clash of conversational implicatures, which is why it would not be said, but it does not mean they cannot occur together as constituents of the same valid inference, nor that the implicatures cannot be cancelled. As we have seen, Hare (1967) gives examples that suggest the inference is legitimate. Satisfiability conditions for imperatives may well serve the role of conversational implicatures in some circumstances.

Concerning the second point, where there appears to be different interpretations of negation, we note that some of these difficulties might be avoided if the ambiguity is carried over into the description of what it is required to satisfy the imperative. For example Type I and II negation with imperatives appears to correspond with (a)telic negation with propositions. Thus the satisfaction of the imperative *"Don't come to the party!"* or *"Don't climb the mountain!"* can both formally be characterised in the same way as the previous example, but with an ambiguity in the description of what it means to satisfy them. In the latter case, either by the addressee *"not climbing (anywhere on) the mountain"*, or by *"not climbing (to the top of) the mountain"*. Formally, any distinction between a Type I and II negation for imperatives will then be parasitic on any treatment of the corresponding classes of negation for indicative negation.

We can now give the formal rules for negation.

(61) Typing (theorem)

$$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{-\, p!_\alpha : \mathsf{Imp}}\, !-\mathcal{F}$$

(62) Satisfaction

(a) $$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \blacklozenge\, p!_\alpha \quad \alpha : \mathsf{Agent}}{\neg \blacklozenge -\, i!_\alpha}\, !-\mathcal{I}$$

(b) $$\frac{p : \mathsf{Pty}_{\mathsf{ag}} \quad \neg \blacklozenge -\, p!_\alpha \quad \alpha : \mathsf{Agent}}{\blacklozenge\, p!_\alpha}\, !-\mathcal{E}$$

It is worth highlighting a minor point about the scope of negation with respect to the implicit future tense. The negation implicitly quantifies over all future eventualities.

(63) *"It is not the case that (in the future) John comes to the party."*
▷ *"Don't come to the party [John]!"*

The negated proposition is not *"(in the future) John does not come to the party."* which might lead to inappropriate results, particularly for actions that can be repeated.

2.7 Conditionals

Conditional imperatives combine a propositional antecedent with an imperative consequent. The intention appears to be that if the antecedent is true, then the imperative consequent is applicable and should be treated in the same way as an unconditional imperative. As an example, the conditional imperative

(64) *"If you see John, say hello."*

suggests that the addressee should *"say hello"* on seeing John. With this example, it seems natural to assume this is applicable the next time John is seen by the addressee (not necessarily every subsequent occasion), and that

it is John who should be the addressee of the salutation. Once again, we make no attempt to analyse these more subtle issues. Any comprehensive analysis of indicatives also has to deal with similar problems, and so there is little point in devising a specific analysis for imperatives. Instead, the idea of the approach is to preserve these finer details of meaning in such a way that any analysis of them in the realm of propositions will automatically apply to imperatives by way of the propositional descriptions of their satisfaction criteria.

There is another aspect concerning what is actually expected of the addressee when uttering a conditional imperative which we do have to consider more carefully. At first sight it might appear perverse to satisfy the previous conditional imperative by seeking to ensure that the antecedent is never true. Certainly it does not appear to be a request to avoid John. However, we can construct examples where the conditional imperative can legitimately be interpreted as such a request.[22]

(65) *"If you see John again, don't even think about coming back."*

In this case, it appears the conditional imperative is a threat that might have something of a propositional flavour (*"if you see John again, you won't be welcome home"*) which can then be used in the addressee's plans when deciding whether or not to see John, or alternatively the example could be interpreted as having an imperative "force" somewhat akin to the threat *"don't see John again [or else]!"* (a disjunctive pseudo-imperative — see Section 2.8).

The question is then whether the apparent difference in attitude to such examples is fundamental to their semantic interpretation, or whether it can be discounted as a "pragmatic issue" that lies outside the realm of the current theory.

There is perhaps an even deeper question; assuming that from the last example we can infer that there is an aspect of the interpretation that corresponds to the proposition *"if you see John again, you won't be welcome home"*, should our semantic theory attempt to capture all such relationships, including the inference of implied imperative content from propositional utterances?

The utterances

(66) (a) *"Wool goes horrible when it is wet."*
(b) *"I think it is going to rain."*
(c) *"Don't you like the jacket I bought you?"*

could all be intended to mean *"Don't wear that awful jumper!"* In this case one would be on fairly safe ground to argue that this is an issue for pragmatics, given the disparity between the form and content of the utterances and the conclusion to which the addressee is intended to come. A similar point is made by Lappin (1982)[23]. Even so, interpreting imperatives in terms

[22] Piwek (2001) discusses similar conditional imperatives where the addressee may have reasons for not satisfying the imperative consequent.

[23] Cf. the notion of "rhetorical role" of Lascarides & Asher (2004).

of propositional descriptions of their satisfaction criteria may be helpful as it provides a bridge between imperative and indicative forms.

One issue that is hard to avoid when considering conditionals is that of modal subordination. This is where the consequent of a conditional appears to be interpreted in a modal context generated by the antecedent (Kratzer, 1981). This is plainly evident in overtly modal examples, such as

(67) (a) *"If you need to go to London, take the train."*
(b) *"If the moon were made of cheese, you would be able to eat it."*

Unfortunately, such modal subordination arises in apparently simple examples, including (64) and (65). In the former case, it seems that the imperative to say hello is only applicable in the future context in which John is seen, the imperative is to be interpreted in the context of the future modality introduced (perhaps implicitly) by the antecedent (see Portner (2006) on this issue).

Once again, we would prefer just to concentrate on those issues that are of direct relevance to imperatives, rather than others, like modal subordination, that are not specific to imperatives and which also arise with indicatives. For this reason, we concentrate on dealing with conditional examples corresponding more directly with the material implication, such as

(68) *"If your name is Peter, say yes."*

In the following, we will assume for our purposes that material implication can capture the relevant notion of conditionality in at least some cases.

When interpreted as an imperative, the conditional expression of the form

(69) $$\phi \rightarrow p!_\alpha$$

is satisfied exactly when either ϕ is false, or $\blacklozenge\, p!_\alpha$ is true. That is

(70) $\blacklozenge(\phi \rightarrow p!_\alpha)$ is true iff $(\phi \rightarrow \blacklozenge\, p!_\alpha)$ is true.

We hypothesise that this is an appropriate pattern of behaviour even where the conditional ($\rightarrow$) is replaced by one that captures the appropriate behaviour for modal subordination, as appears to be present in (64). Within the current setting, the proposed satisfaction criteria is appropriate regardless of whether it is made true by virtue of the fact that ϕ remains false (by way of overt action or neglect) rather than carrying out an action ensures $\blacklozenge\, p!_\alpha$ when[ever] ϕ is true. We will not explore the issue of modal subordination or of generic rules, norms and laws any further here.[24]

Wider pragmatic issues are also ignored here, even if we might wish to consider "desirable" and "undesirable" eventualities to determine whether or not it is appropriate to satisfy an imperative by way of ensuring that the antecedent remains false. Such an analysis of desirable *versus* undesirable — or comparative desirability — does seem to be helpful for pseudo-imperatives (Sections 2.8 and 2.9).

The proposed formal rules governing conditionals are as follows.

[24] Although we will note in Section 6.2 that the use of an example with implicit norms by Jørgensen (1937–38) has perhaps contributed to a not insignificant amount of confusion when it comes to debates over whether imperatives lend themselves to a logical analysis founded on the entailment patterns of indicatives.

(71) Typing

$$\frac{\phi : \mathsf{Prop} \quad p : \mathsf{Pty}_{\mathsf{ag}} \quad \alpha : \mathsf{Agent}}{(\phi \rightarrow p!_{\alpha}) : \mathsf{Imp}} ! \rightarrow \mathcal{F}$$

Note that we might also want $(\phi \rightarrow p!_{\alpha})$ to be interpreted as a proposition if we think it has a purely assertoric use to state a rule that is to be evaluated rather than an imperative.

(72) Satisfaction

(a) $$\frac{\phi : \mathsf{Prop} \quad i : \mathsf{Imp} \quad (\phi \rightarrow \blacklozenge i)\ \mathsf{True}}{\blacklozenge(\phi \rightarrow i)\ \mathsf{True}} ! \rightarrow \mathcal{I}$$

This allows us to satisfy the conditional by seeking to keep ϕ false.

(b) $$\frac{\phi : \mathsf{Prop} \quad i : \mathsf{Imp} \quad \blacklozenge(\phi \rightarrow i)\ \mathsf{True}}{(\phi \rightarrow \blacklozenge i)\ \mathsf{True}} ! \rightarrow \mathcal{E}$$

These rules assume that $\blacklozenge i$ is a proposition, where i is an imperative.

We can also have rules governing *"only if"* constructions. We do not offer a fully worked out theory, but note that such expressions appear to be able to possess both propositional and imperative content. For example,

(73) (a) *"Shut the door only if you are cold."*

(b) *"Shut the door, but only if you are cold."*

(c) *"Bury him only if he is dead."*

(d) *"Bury him, but only if he is dead."*

The first example seems to favour a propositional reading, whereas the second seems to possess a more overtly imperative aspect with a rider, or rather, that there is a stronger permissive element. That is they appear to give overt permission to engage in the action subject to the stated constraint.[25]

Here we ignore such complexities, and just assume that the "only if" construction can be interpreted as a propositional constraint.

(74) Typing

$$\frac{\phi : \mathsf{Prop} \quad p : \mathsf{Pty}_{\mathsf{ag}}}{(p!_{\alpha} \rightarrow \phi) : \mathsf{Prop}} ! \rightarrow' \mathcal{F}$$

(75) Satisfaction

(a) $$\frac{\phi : \mathsf{Prop} \quad i : \mathsf{Imp} \quad (\blacklozenge i \rightarrow \phi)\ \mathsf{True}}{(i \rightarrow \phi)\ \mathsf{True}} ! \rightarrow' \mathcal{I}$$

(b) $$\frac{\phi : \mathsf{Prop} \quad i : \mathsf{Imp} \quad (i \rightarrow \phi)\ \mathsf{True}}{\blacklozenge i \rightarrow \phi\ \mathsf{True}} ! \rightarrow' \mathcal{E}$$

[25] Arguably similar complexities arise with indicatives.

We could attempt to formulate what might be considered pragmatic rules of inference within a variant of the current framework (see Section 4.2). For example, if $\neg\phi$ is more desirable than $\blacklozenge i$ (which we could write as $\neg\phi > \blacklozenge i$) then, pragmatically, $\neg\phi$ is the favoured way of ensuring $\blacklozenge(\phi \rightarrow i)$. If it were known or believed by the speaker that $\neg\phi$ is preferred by the addressee, then utterances of this form could be taken to behave as if the antecedent had been requested. In analysing such behaviour, it might be more appropriate to consider preference orderings over "refinements" of an imperative (Section 3.4) rather than directly over satisfaction conditions. Of course, without constraints on the preference ordering, such a system of rules would run the risk of permitting contradictory conclusions.[26]

2.8 Pseudo-imperatives (or)

Pseudo-imperatives combine an imperative with a future tense proposition. This section will consider disjunctive pseudo-imperatives. Conjunctive pseudo-imperatives are considered in Section 2.9.

It is important to note that the objective of the formalisation given here and in Section 2.9 is not primarily intended to be a definitive account of pseudo imperatives. Rather, the aim is to indicate how such an account can be formulated in a proof-theoretic manner which captures the intuitions of some working in the area, in particular those of Franke (2005a). There may be arguments about whether these are the appropriate intuitions, or whether there is some more general way of addressing such phenomena, but it is to be hoped that any such account will have at least some semantic aspects that can be captured by appropriate rules of proof theory.

Examples of conjunctive pseudo-imperatives include

(76) *"Have another blanket or you will be cold."*

(77) *"Keep walking or I will shoot."*

The first example appears to be an imperative combined with an explanation. The second example appears to be a threat. There seems to be some connection with conditionals

(78) *"If you stop walking I will shoot."*

but with explicitly imperative content. Here the formal theory will allow disjunctive pseudo-imperatives to have a propositional content, whose truth conditions correspond with "conditional" statements of this form, in addition to imperative content with appropriate satisfaction criteria. The truth content of $i \vee \phi$ is thus along the lines of $\neg\blacklozenge i \rightarrow \phi$. This is equivalent to $\blacklozenge i \vee \phi$ in the event that $\rightarrow$ is taken to be material implication. We might question whether this genuinely captures the content of a threat.

[26] If such contradictions where a necessary consequence of a theory of pragmatic inference, then some notion of non-monotonic or defeasible inference would be required (cf. Asher & Bonevac (2005)). There is much existing work in this area of preference and non-monotonicity including von Wright (1963a); Boutilier (1994a,b). Girard (2006) provides a survey of work on preference logic.

There is a question as to whether pseudo-imperatives really have imperative force or whether they just have indicative content. In the case of disjunctive pseudo-imperatives it can be argued that they can have both.

(79) Typing

$$\text{(a)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop}}{(i \vee \phi) : \mathsf{Imp}}\, IoP_i\mathcal{F} \qquad \text{(b)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop}}{(i \vee \phi) : \mathsf{Prop}}\, IoP_p\mathcal{F}$$

Note that ϕ is usually future tense, and assumed, by speaker at least, to be viewed as a negative outcome by addressee.

(80) Satisfaction

$$\text{(a)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad \blacklozenge i\ \mathsf{True}}{\blacklozenge(i \vee \phi)\ \mathsf{True}}\, IoP_i\mathcal{I}$$

$$\text{(b)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad \blacklozenge(i \vee \phi)\ \mathsf{True}}{\blacklozenge i\ \mathsf{True}}\, IoP_i\mathcal{E}$$

(81) Truth

$$\text{(a)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad (i \vee \phi)\ \mathsf{True}}{(\blacklozenge i \vee \phi)\ \mathsf{True}}\, IoP_p\mathcal{E}$$

$$\text{(b)}\ \frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad (\blacklozenge i \vee \phi)\ \mathsf{True}}{(i \vee \phi)\ \mathsf{True}}\, IoP_p\mathcal{I}$$

We cannot use other rules, such as (32)c, to infer anything concerning i True from expressions of the form $(i \vee \phi)$ True as i is not a proposition.[27]

There appears to be a presupposition that the indicative part of a disjunctive pseudo-imperative is to be interpreted as less desirable than complying with the imperative constitutent. Examples where this is not the case sound odd.[28]

(82) *"Have another blanket or you will be happy."*

We have to decide whether this oddness is to be characterised within the semantic theory, or whether it is best considered as a pragmatic aspect of interpretation. It is possible to consider this issue as being no different in kind to hearing a proposition that presupposes something that conflicts with the addressee's understanding of the world. To deal with this semantically would require rules of the form

[27]Some might question in what circumstances expressions of the form $(i \vee \phi)$ and $(\blacklozenge i \vee \phi)$ arise in the interpretation of language. To address this question in full would require at least need an account of the compositional interpretation of language, and perhaps an account of the 'pragmatics' of what we do with such representations. It is quite possible that such clarifications might lead to a reappraisal of the representations and proof rules given here.

[28]Franke (2005a), for example, talks of just desirable or undesirable, but Sonja Eisenbeiss (PC) suggests that this really should be a relative judgment, which matches other proposals concerning preference (see Footnote 26).

(83)
$$\frac{\phi : \mathsf{Prop} \quad i : \mathsf{Imp} \quad \blacklozenge i > \phi}{(i \vee \phi) : \mathsf{Imp}}$$

which indicates that $i \vee p$ is an imperative if satisfying the imperative is preferable ($>$) to the alternative p. This approach mirrors that of Fox (2000) in dealing with presupposition failures with definite descriptors. In that account, representations of sentences containing a non-referring definite descriptor are not provably propositions.

2.9 Pseudo-imperatives (and)

Conjunctive pseudo-imperatives combine an imperative with a future tense proposition by way of a conjunction.

Examples include

(84) *"Close the door and you will become warmer."*

(85) *"Eat the peanut and you will die."*

Initially, these examples may seem to have some imperative content, corresponding with *"Close the door."* and *"Don't eat the nut."* respectively. However, on reflection this is not so apparent. It would appear that some language with imperative case markers (e.g. Hebrew and Greek) do not mark the second case as imperative.[29] If we were to treat these as having imperative content, then the appropriate imperative would once again seem to depend upon the relative preferences of the propositional part and the satisfaction of the imperative part. The argument could be made that a more overtly imperative aspect might be brought to the fore by stressing the imperative constituent, following (73), as in *"Close the door, and you will become warmer."* We will put these issues to one side, and follow Franke (2005a); Russell (2007) and others, and take such statements to be informative propositions with no overt imperative content and with a conditional flavour (see also Culicover & Jackendoff (1997) and Schwager (2005)).[30]

(86) Typing
$$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop}}{(i \wedge \phi) : \mathsf{Prop}}\ IaP\mathcal{F}$$

ϕ is usually future tense. The intended meaning depends on whether the speaker assumes $\blacklozenge i$ is viewed as a negative or positive outcome by the addressee.

(87) Truth

(a)
$$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad (\blacklozenge i \rightarrow \phi)\ \mathsf{True}}{(i \wedge \phi)\ \mathsf{True}}\ IaP_p\mathcal{I}$$

(b)
$$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad (i \wedge \phi)\ \mathsf{True}}{(\blacklozenge i \rightarrow \phi)\ \mathsf{True}}\ IaP_p\mathcal{E}$$

[29] Ron Artstein (PC) and Kakia Chatsiou (PC).

[30] We note in Section 6.3 that this approach may also lend itself to an analysis of counterfactual fiats in vernacular mathematical proofs.

The conditional flavour of conjunction with imperatives is discussed by Russell (2007).

As with the disjunctive pseudo imperatives, there is perhaps a sense in which the desired relationship between i and ϕ is not properly captured by these simple propositional connectives. We will look at this below.

Note that we do not have a typing rule of the form

(88) $$\frac{i : \mathsf{Imp} \quad q : \mathsf{Prop}}{(i \wedge q) : \mathsf{Imp}}$$

This is because we are not treating conjunctive pseudo-imperatives as having overt imperative content, even if the speaker intends that the addressee responds or acts in some particular way as a consequence of understanding the propositional content of the pseudo-imperative.[31]

We could have such a rule, and corresponding satisfication criteria, if we thought that such utterances always had imperative content. Alternatively, if we believe that we should model the imperative content semantically in the case where the outcome ϕ is preferred in some way (at least preferred over not engaging in an action that satisfies the imperative i) then we can use preferences as felicity conditions on the typing of such utterances, along the following lines.

(89) $$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad \phi > \neg \blacklozenge i}{(i \wedge \phi) : \mathsf{Imp}}$$

This could then be given a satisfaction criteria that is also conditional on the case where the pseudo-imperative is judged to have imperative content.

(90) (a) $$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad \phi > \neg \blacklozenge i \quad \blacklozenge i \ \mathsf{True}}{\blacklozenge(i \wedge \phi) \ \mathsf{True}}$$

(b) $$\frac{i : \mathsf{Imp} \quad \phi : \mathsf{Prop} \quad \phi > \neg \blacklozenge i \quad \blacklozenge(i \wedge \phi) \ \mathsf{True}}{\blacklozenge i \ \mathsf{True}}$$

We will leave it to the reader to judge the merit of such an approach.

3 Practical Inferences and Refinement

Here we consider some traditional criticisms of accounts of logics of satisfaction for imperatives. The general argument is that some satisfaction inferences, particularly those governing disjunction, are counter-intuitive, and allow us to infer things that have not been commanded. We will review these arguments, and claim that the "intuitive" patterns of entailment that are perhaps best characterised by way of *refinement* relations.

Before looking at these criticisms in detail we consider the notion of practical inference as it arises with *modus ponens*, arguably the least problematic of such inferences (Ross, 1941).

[31] In this case, we take this aspect of the interpretation to belong to pragmatics rather than semantics as such. See Section 4.2.

3.1 Practical Modus Ponens

Examples of practical inferences that combine propositions with imperatives were given in (20), repeated here,

$$\text{(20)} \qquad \frac{\textit{“If someone is ill, give them an injection”} \quad \textit{“John is ill”}}{\textit{“Give John an injection”}}$$

A similar example is given by

$$\text{(91)} \qquad \frac{\textit{“If someone is ill, give them an injection”} \wedge \textit{“John is ill”}}{\textit{“Give John an injection”}}$$

where the premises are explicity conjoined.

A number of questions spring to mind.

(a) What kinds of premises and conclusions can be combined in this way?

(b) When combining mixed premises, i.e. propositions with imperatives, how should such expressions be typed and evaluated?

(c) How do we make the relevant kinds of inference?

The issues of (b), types, and (c), inference, perhaps go together, as the typing of expressions dictates the kind of inferences that one might expect to be able to express. We might also be concerned about whether (20) is really an inference in the sense we have been using, or something else, such as a refinement (Section 3.4). Here, we will simply note that if the practical inference syllogism is annotated explicity with notions of truth (of propositions) and satisfaction (of imperatives), then the proposed logic of satisfaction goes someway towards providing an account of this particular form of practical inference. Adding appropriate annotations to (20) yields an inference of the following form.[32]

$$\text{(20}'\text{)} \qquad \frac{\blacklozenge(\textit{“If someone is ill, give them an injection”})\ \mathsf{True} \quad (\textit{“John is ill”})\ \mathsf{True}}{\blacklozenge(\textit{“Give John an injection”})\ \mathsf{True}}$$

Once we have the argument in this form, we can account for it using (72)b and (34)b without appeal to any other notion of entailment: if it is true that the conditional imperative is (to be) satisfied, and that its antecedent holds, then it is true that the imperative in the conclusion is (to be) satisfied. Moreover, failing to *“Give John an injection”* would be at odds with any undertaking to satisfy the conditional imperative. There may be outstanding issues concerning the proper analysis of undertakings, and how such inferences involving satisfaction are to be employed when an agent is deciding what to do. In this context it becomes apparent that there are other entailment patterns that are worth exploring. One such alternative is the notion of entailments of *commands*: the question is then, if an imperative has been commanded, what

[32] This can be contrasted with the proposal of Beardsley (1944) (Section 6.4, example (104), where the inference is over the desires of the speaker, rather than the satisfaction criteria.

other commands are a consequence of it?[33] Others have made appeal to this notion, including Lewis (1972), Segerberg (1990) (Section 6.5) and others.[34]

3.2 Ross' Paradox

It is often observed that adopting a rule corresponding to the disjunction introduction rule may give rise to difficulties when it comes to justifying actions by appeal to an imperative. Ross (1941) was perhaps the first to raise this issue, and it has subsequently become known as Ross' Paradox. We will discuss this point briefly and indicate why we don't think it should be considered a major concern.

The "paradox" is founded on a couple of assumptions.

(a) Imperatives are involved in entailment relationships that mirror those for propositions.

(b) An individual is obliged to carry out the entailments of an imperative. (Or even, an individual is *justified* in carrying out actions that satisfy the entailments of an imperative).

The argument then proceeds as follows. As p implies $(p \vee q)$, so $p!$ implies $(p! \vee q!)$ [from (a)]. But then, performing an action that satisfies $q!$ may then be justified by appeal to $p!$ — *for any* $q!$*, and any* $p!$ [from (b)]. From this, the argument goes, we should conclude that not all aspects of imperatives are amenable to logical rules. The force of the argument is perhaps made more strongly if we interpret the imperative $p!$ as meaning *"Make it the case that* p*"*.[35]

An alternative conclusion is that "justification" does not work this way, in the same way that we are not justified in saying that q is *true* just because $p \vee q$ follows from an assertion that p is true.[36] Furthermore, it could be argued that the paraphrase of an imperative as being a statement of the form *"make it the case that . . . "* might be a misleading and inappropriate reduction from imperatives to propositions.

It is also worth noting that this particular aspect of Ross's argument does not by itself rule out the possibility of a logic of satisfaction, but rather is against the possibility of a logic of satisfaction that is *simultaneously* a logic of justification (or *validity*, in the terminology of Ross). We will discuss a specific objection he raises against such logics below.

In the present theory, we would replace "p is true" — or rather $p(\alpha)$ True — by the satisifaction statement $\blacklozenge\, p!_\alpha$ True. On a free choice reading, it would follow that $\blacklozenge(p \cup q)!_\alpha$, but it would not follow that $\blacklozenge\, q!_\alpha$.

It is perhaps worth looking into the background behind the claim that a logic of imperatives is problematic.

[33] This is what Ross describes as inferences of "validity" (Ross, 1941).

[34] Arguably it is these different possibilities for relating imperatives to propositional that have been the source of some confusion in the literature. The problems described by Ross (1941) are essentially the difficulties of reconciling the inferences of satisfaction and validity. (Section 3.2).

[35] See Kenny (1966) for example.

[36] Indeed it may be possible to devise a form of Ross' paradox for propositions, if suitable assumptions are made.

Ross considers several solutions to the problem of practical inference, focusing on the issue of *satisfaction* (when do we judge that an imperative has been fulfilled) and *validity* (what is it that is understood to have been ordered). It is perhaps in this sense that he deems disjunction introduction to be "invalid" in that it does not then conform to what is understood to have been ordered.

Ross advances the view that

(92) "... the characteristic feature of the *existing practical inferences is that they purport to bring about a combination of the results to which the logic of satisfaction and the logic of validity may lead respectively*, namely so that the transformation rules of the logic of satisfaction are complied with, but that relevance with regard to the validity of the imperative is ascribed to the transformation." Ross (1945), author's emphasis.

He then goes on to show that in most cases, except for negation and subsumption, it does not appear possible to reconcile the logic of satisfaction with the logic of validity. Furthermore, he argues that even in the case of subsumption and negation, the inferences make appeal to tacit assumptions concerning the notion of validity (for example that it is is not possible for someone to demand that A and its negation both hold).

Notice however that if we do not accept Ross's hypothesis (92) — that is that notions of validity and satisfaction must be applied *together* when determining legitimate practical inference entailment patterns — then his argument no longer applies. The alternative hypothesis that we offer in its place is that, while agreeing that there are at least two distinct notions (which we term *satisfaction* and *refinement* (Section 3.4) that are relevant when reasoning with imperatives and when deciding how to comply with imperatives, these notions can be formulated without making an *a priori* assumption that every individual entailment or inference rule must *simultaneously* encapsulate both notions in order to characterise practical entailments.

We can demonstrate that there is a problem with the claim advanced in (92) that practical inferences are required to combine the results of a logic of satisfaction with those of a logic of validity. Consider a formal system that allows us to reason with the ways in which a computer program might implement a specification. It is reasonable to suppose we would wish to be able to make the following inferences.

(93) (a) Given the need for a program to meet the requirement $(a \vee b)$, this can be achieved by making it meet the requirement a.

(b) Given a program satisfies the requirement a then it also satisfies the requirement $(a \vee b)$.

In the case of (93a), $(a \vee b)$ can be "refined" to a, so we have rule of the form

$$\ldots (a \vee b) \ldots \longrightarrow \ldots a \ldots$$

In the case of (93b) from the satisfaction of a we can infer the satisfaction of $(a \vee b)$, so we have rule of the form

$$\ldots a \ldots \longrightarrow \ldots (a \vee b) \ldots$$

There is a connection between these two notions, and it is only right for the disjunction itself to have a consistent interpretation in both cases. There is not, however, any clear case for requiring that the rules we are following in (93a) and (93b) be identical in form and interpretation.[37] Indeed, it is not immediately clear how this could be achieved.[38] The fact that we *can* consider inferences of the form (93a) without having to combine it directy into a single rule with inferences of the form (93b) suggests that the assumptions of (92) are incorrect, especially if we view programs-implementing-specifications as involving a form of practical inference.

If there is any doubt about this last assumption, then an alternative approach would be to present a formal account of practical inference which does not require individual rules to encapsulate both satisfaction and "validity". Such an account is sketched in Section 3.4.

So the arguments can be summed up as follows: if (92) is the correct characterisation of practical inference, then, according to Ross (1945) there can be no formalisation of such a logic. The alternative is that if there are such formalisations, then the characterisation of (92) is incorrect. The contention made here is that there such formal systems are already used to reason about computer programs and their specifications, among other things, and so the latter characterisation appears to be the most appropriate.

Let us turn to Ross's specific arguments against logics of satisfaction, rather than practical inferences as such. He argues that logics of satisfaction such as the one proposed by Hofstadter & McKinsey (1939) gain nothing, in that imperatives just appear to be a syntactic decoration of propositions. This view is independent of the hypothesis (92). This may indeed be the case with Hofstadter & McKinsey (1939), but that is arguably merely contingent on the precise formulation. If, for example, such logics have behaviours that capture insights into the meaning of imperatives and fiats then surely something has been gained, especially if such behaviours diverge from that of a naïve propositional re-interpretation. In the case of vernacular proofs, if the fiat *"Let f be a continuous function."* is to be interpreted as giving rise to some notion of hypothetical conditionality, along the lines of Section 2.9, then surely something has indeed been gained.

Perhaps the real issue behind Ross' Paradox is a desire to see inferences involving imperatives as being a way of determining an answer to the question *what should be done?* in the face of discourse involving imperatives. That is to develop a theory of *practical inference* or *satisfactoriness* relationships, in the terminology of Kenny (1966).

3.3 Kamp's Free Choice

According to Kamp (1973), the common perception is that inferences involving Free Choice disjunction should actually go in the other direction to that suggested by following the pattern of classical logic. That is, from

[37] That is, there is no obvious case for asserting that the notions of refinement and satisfaction be *equated*.

[38] We could of course adopt one rule and interpret it in two different ways, but this would not really be in the spirit of (92).

(94) *"swim or run!"*

we should be able to "infer" one of the following

(95) (a) *"swim!"*
(b) *"run!"*

On this issue, it might be worth considering the notion of *refinement* from computer science (Wirth, 1971). Essentially a specification, program or data structure is refined by making it more specific, whilst keeping the relevant behaviour. There is a connection between a (correct) refinement relationship and an entailment relationship. A specification (or program) A' is a correct refinement of A (formally, $A \geq A'$) if the behaviour of A is included in the behaviour of A' ($A' \vdash A$). Presenting this in a style that is sympathetic to the current paper, if a program p satisfies the refined specification A', then it will also satisfy A, so $A \geq A'$ when $\blacklozenge A' \vdash \blacklozenge A$.[39] Considering (94) and (95), and eliding some of the typing conditions, we can see that

(96) *"Swim or run!"* $\geq$ *"Swim!"*

or more generally

(97) $$(p \cup q)!_\alpha \geq p!_\alpha$$

is a legitimate refinement *because*

(98) $$\frac{\blacklozenge p!_\alpha \ \ \mathsf{True}}{\blacklozenge (p \cup q)!_\alpha \ \ \mathsf{True}}$$

This seems to be a plausible explanation for the feeling that the there is legitimate way of deriving one of (95) from (94), it is just that the relationship in question is one of refining the specification of what could be done to comply with a request, rather than necessarily being an entailment relationship in the logical sense. The idea of refinement is discussed a little more below, in Section 3.4.

Of course, this does not entirely deal with the issue of practical inference, where we wish to reason with combinations of propositions and imperatives, as in (20) which was picked up in Section 3.1.

3.4 Refinement

Although we have no intention of presenting a complete refinement calculus here, as this is not the main focus of the current paper, it is perhaps worth sketching some sample rules that appear relevant to examples such as (96).

(99) Some plausible rules of refinement:

(a) **Disjunction:** $i \leq (i \vee j)$, and $i \leq (j \vee i)$
This strengthening supports the intuitive derivation for disjunction, as exemplified by (96)

[39] Note that this relationship perhaps should not apply in the other direction. Consider that if $\bot$ is an inconsistent statement, then $\bot \vdash A$, for any A, but we probably would not wish to say that $A \geq \bot$.

(b) **Conjunction:** $(i \wedge j) \leq i$ and $(j \wedge i) \leq i$
It is acceptable to do more than is asked (provided that compliance with j is not otherwise prohibited)

(c) **Conditional:** If p true, then $i \leq (p \rightarrow i)$
For the case when the antecedent of a conditional imperative is true. This might be redundant, depending on the impact of the rules of inference on the refinement relation.

(d) **Identity:** $i \leq i$
Imperatives are refinements of themselves.

(e) **Transitivity:** $(i \leq j \wedge j \leq k) \rightarrow (i \leq k)$
Refinement of imperatives is transitive.

In all cases, i, j : Imp and p : Prop.

Perhaps unsurprisingly, this could be formulated as a logic, for example as a sequent calculus along the following lines.

(100)

$$\text{(a)}\ \frac{\Gamma \leq i, \Delta}{\Gamma \leq (i \vee j), \Delta} \vee_r \quad \text{(b)}\ \frac{\Gamma, i \leq \Delta}{\Gamma, (i \wedge j) \leq \Delta} \wedge_l \quad \text{(c)}\ \frac{\Gamma, [p] \leq j, \Delta}{\Gamma \leq (p \rightarrow i), \Delta} \rightarrow_r$$

$$\text{(d)}\ \frac{}{i \leq i}\, I \qquad \text{(e)}\ \frac{\Gamma \leq \Delta, i \quad i, \Sigma \leq \Pi}{\Gamma, \Sigma \leq \Delta, \Pi}\ \text{Cut}$$

This could be extended to include the corresponding left (right) rules for (a), (b) and (c), together with rules for negation.[40]

$$\text{(a')}\ \frac{\Gamma, i \leq k \quad \Sigma, j \leq k}{\Gamma, \Sigma, (j \vee i) \leq k} \vee_{l \text{ intuitionistic}} \quad \text{(b')}\ \frac{\Gamma \leq i, \Delta \quad \Sigma \leq j, \Pi}{\Gamma, \Sigma \leq (i \wedge j), \Delta, \Pi} \wedge_r$$

$$\text{(c')}\ \frac{\Gamma \leq [p], \Delta \quad \Sigma, i \leq \Pi}{\Gamma, \Sigma, (p \rightarrow i) \leq \Delta, \Pi} \rightarrow_l$$

$$\text{(f)}\ \frac{\Gamma \leq i, \Delta}{\Gamma, \neg i \leq \Delta} \neg_r \quad \text{(f')}\ \frac{\Gamma, i \leq \Delta}{\Gamma \leq \neg i, \Delta} \neg_r$$

Where i, j, k are imperatives, p is a proposition, and $[p]$ is used to denote propositions that are true in the context.

We could consider such a logic of refinement as characterising a notion of *justification*. The formalisation given here is not a complete analysis. For example, we have also not considered pseudo-imperatives. We might also wish to have rules that allow a Free-Choice disjunctive imperative to be refined only to a specific disjunct if the other disjunct is not satisfiable. Where there is a genuine choice, a preference ordering ($>$, as mentioned in Sections 2.7, 2.8 and 2.9) might be used to decide which refinement is to be adopted.

[40] For simplicity, we shall not present the structural rules or all the permutations of the logical rules.

In producing a more detailed and precise theory of refinement for imperatives, other issues might also have to be considered, such as whether it is appropriate to express refinement in terms of the imperatives themselves, rather than their satisfaction conditions, for example. Even some of the given rules might be contentious or problematic, both empirically and formally, and whether or not they are appropriate may depend upon the precise details of the satisfaction relationship.

In case the foregoing discussion remains unclear, we can consider the parallel case with indicatives. A set of propositional sentences constitutes a *theory*. We can do various things with such a theory. One is to appeal to a truth conditional notion of entailment to derive theorems of the theory. We might also try to determine whether the theory is *consistent*, and try to find some concrete way of satisfying the theory. In the latter case, we might consider a `Prolog` program. When we run such a program we are effectively trying to find an instantiation of variables etc. that satisfy the statements that constitute the `Prolog` program. It is truth-conditionally the case that $p \vee q$ follows from p, but when running a program we would try to satisfy a statement corresponding to $p \vee q$ by trying to find a way of satisfying either p or q. This "inference" from $p \vee q$ to p or to q is not seen to undermine the entire truth-conditional framework of propositional logic. Why should similar observations in the case of imperatives be thought to undermine the case for a satisfaction-conditional logic? Refinement can be thought of as generalisation of the kind of reasoning that is at work when running a `Prolog` program.

It should be clear that it is possible to produce a logic of practical inference which supports intuitively appropriate behaviours for both subsumption and disjunctive, for example. Furthermore, contrary to the arguments of Ross (1945) and others (Section 3.2), it should be apparent that there is no need for such patterns of behaviour to simultaneously encapsulate the satisfaction relationship. There are of course close relationships between the proposed logics of satisfaction and refinement, but even without exploring this in detail, it should be apparent that such a relationship need not be of the form of identity of inference rules and their interpretation for the two notions.

4 Alternative formalisations

In this account we have given what might be described as a functional account of satisfaction; $\blacklozenge\, p!_\alpha$ is the propositional criteria that describes what must (become) true for $p!_\alpha$ to be satisfied. In earlier versions of this work, a relational approach was adopted, where the statement $\phi \triangleright p!_\alpha$ indicated that the proposition ϕ satisfied the imperative $p!_\alpha$. In the atomic case, we would have $p(\alpha) \triangleright p!_\alpha$. On such an account it might also be necessary to distinguish between this basic notion of satisfaction, and *derived* satisfaction (where $\psi \triangleright p!_\alpha$ if $\psi \vdash p(\alpha)$). The functional approach was adopted in this paper as it appears amenable to a more straightforward formal presentation.

One reason for noting the possibility of a relational presentation is that

it may contribute to an intuitionistic interpretation. Within an intuitionistic framework, a proposition is "true" if a proof, or *witness*, can be found for it. Such systems have been used to give an account of indicatives and discourse (Ranta, 1994; Dávila-Pérez, 1995). This "relational" view of truth of a proposition echoes a relational view of satisfaction of imperatives. It could be fruitful to explore whether the two can be combined in an informative manner. It is worth observing that the formalisation of conjunction used in an intuitionistic interpretation possesses a conditional character, which might be exploited when accounting for conjunctive pseudo-imperatives, for example.

Hare (1949) suggests that the 'logical' patterns of behaviour of indicatives and imperatives comes from the behaviour of a notion that underlies both. In principle it should be possible to reformulate the current theory to capture this idea.

Essentially we can consider a category of 'proto-expressions' (or 'phrastics', in the terminologhy of Hare (1952)) which can be combined with connectives to form more complex expressions. These can then be augmented by 'dictive functions' (or 'neustics') to form either propositions or imperatives.

Perhaps this is not radically different from the current formulation, where agentive properties ($\mathsf{Pty}_{\mathsf{ag}}$) arguably play a role not too different from that of these proto-expressions. One difference is that the logical connectives between propositions (i.e. $\wedge, \vee, \neg$), for example, would perhaps have to be defined in terms of operators between $\mathsf{Pty}_{\mathsf{ag}}$ (i.e. $\cap, \cup, -$) rather than there being mere equivalences between expressions involving these operators.

Another alternative would be to consider giving an algebra for imperatives, including rules such as $(i \wedge i) \equiv i$ and $(i \vee i) \equiv i$, and so allowing us to simplify and find relationship between imperatives directly, rather than always having to express things in terms of satisfaction criteria. Such rules have not been considered here because the focus has been primarily on devising an explicit logic of satisfaction that, in particular, allows us to consider and address the problems identified by Ross (1941) and others, and avoid the difficulties presented by theories of action, and somewhat unsatisfactory appeals to Gricean maxims.

4.1 Preconditions and Satisfiability

In the rules governing entailment patterns over satisfaction conditions we do not consider the preconditions of actions; or more precisely, what must hold in order for it to be possible to satisfy an imperative. That is, we do not consider the *satisfiability* of imperatives.[41] This is because the rules governing the satisfaction conditions are merely concerned with what *has* to be true, not what *can* be true. Complying with the imperative i requires ensuring that $\blacklozenge\, i$ is the case. If it is not possible to make $\blacklozenge\, i$ the case, because its preconditions are not satisfied for example, then we could argue that corresponds to saying

[41] We use the term "satisfiability" where others have used "validity". This is to avoid confusion with alternative, although possibly related uses of the term "validity" in the literature on imperatives. Other authors have used the term "correctness" for this notion. Again, we which to avoid this expression due to possible confusion with the use of the term "correctness" in the literature on refinement.

that the preconditions for i itself are not satisfied.

We can devise rules that allow us to determine the preconditions of complex imperatives in terms of the presuppositions of constituent satisfaction conditions. In the event that we also had a theory of temporal sequencing, we could also consider cases where compliance with one imperative actually ensures that a subsequent imperative is valid, as in

(101) *"Open the door and keep it open!"*

This correspondence with van Eijck's notion of entailment (van Eijck, 2000). If we wish, we can then amend the rules governing satisfaction, so that satisfaction is conditional on satisfiability.

The argument can be made that this is a redundant exercise: the way we have conceived of satisfaction conditions implicitly deals with issues of satisfiability. An imperative is satisfied if some appropriate propositional description is true at some point in the future, but it can only be true if the presuppositions of that propositional description hold, and it is precisely the presuppositions that capture the satisfiability conditions of the imperative.

There is a potential flaw in this argument, concerned with timing. This can be illustrated with the simple imperatives considered here. In these cases, the expectation is that the imperatives will be complied with as soon as possible. Under the above proposal, there is a possibility that the presuppositions of the satisfaction conditions do not hold at the time of the utterance of the imperative, thus the imperative might not be "valid" when uttered. If this is thought problematic, then, in the case of atomic imperatives at least, the preconditions for complying with $p!_\alpha$ are the presuppositions of $p(\alpha)$, rather than the presuppositions of $\diamond p(\alpha)$. On this view, the implicit, "built-in" notion of satisfiability would not be adequate by itself.

As has already been noted (Footnote 20) presupposition and accommodation may play a crucial role in the meaning of imperatives, particularly in the case of free-choice disjunction. For this reason, it merits further study.

4.2 Pragmatic Interpretation

This paper is not primarily concerned with pragmatic interpretation. We take pragmatics to be concerned with the relationship between an speaker, addressee and utterance, in particular, with the question of how an addressee would reasonably respond to a given utterance, and how a speaker might intend an addressee to respond.

Pragmatics may cover cases were the expected response is at odds with the apparent form of an utterance, as in (66) (page 22), but we take it that it is also concerned with utterance processing issues. In the case of propositions, a semantic analysis is usually restricted to the notion of truth conditions and inference, whereas a pragmatic analysis may be concerned with belief update and revision. With imperatives, such a pragmatic analysis might focus on how a given imperative is to be processed: under what circumstances is it to be complied with; how is it complied with; and how conflicts with competing concerns are to be resolved.

The parallel with propositions is helpful, in that it suggests it is appropriate to abstract away from pragmatic concerns, but the boundary is a subtle one. We might consider models that focus on addressees' processing of imperatives to be pragmatic in nature. Such formalisations may model imperatives in terms of "commitments" or "to do" lists (Piwek, 2000; Portner, 2005) and others). This is in the spirit of Kenny (1966), in which he says "We work out, with the aid of logic, not what is the case but also what we are to do." However, if this is what we mean by pragmatics, then there is a sense in which the discussion of practical inference in connection with conditional (Section 2.7 and 3.1), refinement in connection with disjunction (Section 3), and preference issues that arise with pseudo-imperatives (Section 2.8 and 2.9) are all touching on pragmatic issues.

One area of pragmatics which has not been mentioned so far concerns the imperative parallel of belief revision. Imperatives from an appropriate authority may implicitly grant permission to break some background obligations. There may then be questions as to which obligations might be broken (Kamp, 1973; Asher & Bonevac, 2005). Although permissions and obligations may be considered to lie in the realm of non-propositional statements, there are clear parallels with many issues that arise with propositional statements in the context of belief revision.

5 Formal Properties

Given a formal systems such as the logic of satisfaction presented here, it is possible to determine whether it possesses a range of properties including *cut elimination*, *sub-formula property*, *normalisation* results and *local consistency*. We do not propose to pursue these important issues here as the main objective of the paper is to demonstrate that it is at least in principle possible to have a meaningful logic of satisfaction that is more than a mere translation into propositioinal logic.

A model for the core theory with imperatives, conditionals and pseudo-imperatives is sketched in the appendix. Essentially the idea is to interpret expressions in the theory in some "uninterpreted" language, and then provide functions that model the type membership and truth judgements of the theory.

There is a sense in which the model does not provide additional enlightenment as to the intended interpretation of expressions in the theory. Given the "representationalist" approach taken, this is not necessarily a criticial issue; the main point is to go about demonstrating the kinds of structural constraints any model would have to support in order to validate the inferences of the theory.

In principle it should be possible to check which structural constraints are supported by an alternative theory or model, so as to determine which of the proposed inferences given here are validated.[42]

[42]In practice, we may conjecture that this may be complicated by certain theory-specific features that render the relevant structural behaviour somewhat opaque.

6 Comparison with other theories

Comparison with other theories can be made on a number of grounds. These include the theoretical machinery that is used (e.g. actions, possible worlds etc.) and on the view of what kind of relationship is being captured between imperatives. The latter question is often resolved by consideration of the attitude towards Ross' Paradox. The issue of empirical adequacy is a primary concern, but cannot be considered without reflecting on the nature of the relationships that a theory is seeking to capture.

There are many proposals for the semantics and pragmatics of imperatives, and discussions of how best to construe and express inferential behaviour. It is not possible to do full justice to all of this work here. Instead we just focus on a small selection proposals, criticisms and frameworks. We do not consider pragmatic accounts of meaning here, such as those of Piwek (2000) and Portner (2005) (see Section 4.2).

6.1 Propositions, Imperatives and Actions

The idea of considering some "propositional content" of imperatives is not new (e.g. Jørgensen (1937–38); Hare (1949); Stenius (1967); Lewis (1972)). It is even described as "the standard approach" by Huntley (1984). Furthermore, others have also thought to include agent/addressee arguments in their accounts (Portner, 2005). However, it would appear that such a proposal has never really been presented in full detail, or with full consideration of para/pseudo-imperatives and conditional imperatives.

Note that here we are concerned with a non-reductive analysis of propositional content; it is not the objective of this work to find ways of merely *reducing* imperatives to a proposition, with all of the problems that follow, nor of expressing propositional content in terms of necessary and sufficient conditions for the speech act in question, but rather consider imperatives as things in themselves that are related to some propositional "content" by way of satisfaction or "fulfilment" criteria (Hare, 1967; Lappin, 1982) akin to the "outcomes" of Ginzburg & Sag (2000). The relevant propositional description of the satisfaction criteria is not arbitrary, but can be determined from the content of the imperative in question.

Some accounts include a specific notion of an action which is intimately related to a proposition (Segerberg (1990); Lascarides & Asher (2004); Pérez-Ramírez & Fox (2003a); van Eijck (2000) and others). The imperative is then a request to perform this action in order for the underlying proposition to become true.

If we admit actions as such into our account, we need to ask ourselves what actions are (at least for the purposes of imperatives). A working assumption is that they are physical things that are performed in the world which lead to a change. They may typically be sequenced, decomposed, repeated etc. There are open questions about how far they might be decomposed. One issue of concern is the nature of intent. As has been observed before, a person who hangs a piece of bread in the water on a hook can only be described as engaging in the act of fishing if that is what she intends to

do, regardless of whether or not she catches fish (Hamblin, 1987). This is related to the notion of decomposition of actions. There may be many ways of decomposing the activity of fishing into constitutent actions, but it is the intent that is really critical.[43] Even if we disregard intent, and consider only extensional behaviour, in general there is no way of giving a sufficient analysis of the activity by reduction to "primitive" actions (there may be ways of fishing that have not yet been developed). Even without a detailed anaylsis, it can be seen that actions raise a number of difficult problems.

In the current account, we avoid such problematic issues by abstracting away from this notion of an action, and instead we concentrate on the state of affairs that results in the post conditions. However, these are not postconditions that merely describe the resultant state of affairs (e.g. that the door is closed), rather they state that the agent in question engages in the relevant action or activity, or not (e.g. that John closes the door). Indeed, it is not clear what purpose is served by considering actions as such (rather than descriptions of actions) *if the primary objective is to understand the inferential behaviour of imperatives themselves, rather than contingent facts about actions.* We can go a long way without having to face questions about actions, causality, events, intentions, the frame problem (McCarthy & Hayes, 1969) and the relationship between actions and events (e.g. see Bach (1980)).

6.2 Jørgensen: imperative inferences

Jørgensen's account of imperative entailment patterns (Jørgensen, 1937–38) is worth considering not because it is the first or the most complete, but because arguably it gave rise to the arguments of Ross (1941) concerning whether imperatives to support the same patterns of entailment as propositions.

The phenomena of concern were syllogisms containing imperatives, what others have called "practical inferences" (Kenny, 1966) (see Section 3.1).

Essentially Jørgensen (1937–38) offers the view that imperatives combined an *imperative factor* — the actual desire or wish on the part of the person who uttered the imperative — and an *indicative factor* — the content of the imperative that describes what it is that is desired or commanded. The argument is that the indicative factor is propositional in nature, and thus supports the same entailment patterns as indicatives. Jørgensen deemed that the imperative factor is an issue of psychology, and hence outside the bounds of a logical theory.

To some extent this parallels the view taken in the present paper. The indicative factor in this case being the satisfaction conditions of the imperative. In Jørgensen's case there was a slight complication. Initially he takes the appropriate content to be something concerned with the desired outcome (not far removed from the notion of satisfaction). However, he makes a rather significant and unfortunate revision to this proposal which undermines his

[43] There is also the thorny issue of unintended consequences of actions that contribute to an intended action.

thesis. When considering the inference of the form

$$\frac{\textit{“Keep your promises”} \quad \textit{“This is a promise of yours”}}{\textit{“Keep this promise”}}$$

he initially “translates” this into the following propositional argument

$$\frac{\textit{“All promises are to be kept”} \quad \textit{“This is a promise [of yours]”}}{\textit{“This promise is to be kept”}}$$

Jørgensen then observed that it is difficult to see how to verify a sentence of the form *“All promises are to be kept”*. For this reason he goes on to suggest that the propositional content of “such and such action is to be performed” are an abbreviation of “There is a person who is commanding that such and such action is to be performed” (Jørgensen, 1937–38).[44]

There are some crucial observations that can be made about this example, and the revised characterisation of the indicative content of imperatives. One is that the problem of verification that Jørgensen identified in the example is at least in part due to the normative nature of the example. The interpretation of normative expression does deserve some serious consideration, but that would be a substantial digression, and problems in their analysis are not confined to imperatives.[45] Instead we will observe merely that the work-around which is intended to address the problem of obtaining verifiable indicative content from a normative universal imperative appears to undermine the thesis that Jørgensen is trying to make.

The problem lies in making the indicative factor correspond to overt descriptions of the commands that have been issued, rather than what would satisfy the commands that have been issued.[46] Overt descriptions of what is commanded do not follow the patterns of entailment of indicatives in the way that Jørgensen would like. If A is commanded, it does not mean that $A \vee B$ is commanded. By switching from his initial proposal that indicative content is something to do with descriptions of outcomes to descriptions of commands, Jørgensen renders himself open to the argument of Ross (1941), that imperatives should support inferences akin to disjunction introduction, but when characterised as descriptions of what is commanded, they clearly should not. This issue was explored above in Section 3.

6.3 Hofstadter and McKinsey: a logic of satisfaction

Hofstadter & McKinsey (1939) present a comprehensive theory of the logic of satisfaction for fiats. Fiats are akin to imperatives except there is no agency, as in

(102) (a) *“Let the door be shut!”*

[44] This particular view of the meaning of an imperative is echoed in a later proposal of Lewis (1972).

[45] One possibility for dealing with such inferences is to consider them as expressing some entailment relationship involving hypothetical satisfaction of norms and imperatives.

[46] A switch between *satisfaction* and *validity*, in the terminology of Ross (1945).

(b) *"Let f be a continuous function."*

Their satisfaction criteria are then merely the statement of fact that the door is closed, or that f is a continuous function. If the focus is on mathematical vernacular, then the satisfaction of such fiats could be assumed to be immediate.

Hofstadter & McKinsey remark that Jørgensen's apparent view that "there seems to be no reason for, indeed hardly any possibility of, constructing a specific 'logic of imperatives"' stems from a failure to appreciate that more than one verb can be in imperative mood within a single sentence.

Unfortunately the focus on fiats, and mathematical statements, means that it is possible to devise a simple translation from fiats to propositions that ignores many subtle issues. Indeed, the translation can be so simple that it is possible to take fiats as mere syntactic decorations of propositions. Beardsley (1944) argues against the presumption that statements should be formally assumed to be more basic than imperatives, and Ross (1941) has argues that such a translation is not informative.

The logic of satisfaction proposed in the current paper is not based upon a mere translation from imperatives to propositions, and takes account of additional data, such as pseudo-imperatives. Indeed, the treatment of conjunctive pseudo-imperatives in Section 2.9 lends itself to an analysis of fiats in the mathematical vernacular where the validity of the proof is dependent on a (perhaps hypothetical) assumption that the fiat is satisfied. This appears to lend itself to analysis of counter-factual fiats, as used in proof by contradiction, as in *"Let m be the largest prime number"*.

6.4 Beardsley: imperatives and indicatives

Beardsley (1944) makes a number of comments about some then contemporary proposals. For example, she says that it does not seem right to assume that propositions are somehow more basic than imperatives (as in Hofstadter & McKinsey (1939), for example). She argues that in accounts like that given by Jørgensen (1937–38)) it is not right to view inferences between propositional correlates of imperatives as meaning there are genuine inferences between imperatives themselves. Rather, the inferences are between expressions concerning the *satisfaction* of imperatives.[47] The former point appears to address some of the criticisms raised by Ross (1941).

With the inference (cf. Section 3.1)

(103)

$$\frac{\textit{"If it is raining, hand me the umbrella."} \quad \textit{"It is raining"}}{\textit{"Hand me the umbrella."}}$$

Beardsley (1944) claims this should really be interpreted as

[47]She also appears to take the view that satisfaction of an imperative should not be considered to be something in the present tense.

(104)

"If it is raining, she want me to hand her the umbrella." *"It is raining"*

"She wants me to hand her the umbrella."

This is clearly somewhat different to the intended interpretation of the logic of satisfaction as described in this paper. It is instead more akin to a logic of validity, in the sense of Ross (1941) (Section 3.2): it reflects the reasoning of an agent about the mental state of the commanding authority. Indeed Beardsley argues that the a crucial aspect of the meaning of an imperative is the relationship between individuals to the imperative, just as in the case of indicatives there is a relationship there is some *assert* relation between what we might call the predicative content of the indicative and the person who utters it, where 'to assert' means to desire that others believe the indicative content. For both indicatives and imperatives, this agent focused meaning does not support disjunction introduction.

Beardsley also discusses cases of conditional imperatives and what would call disjunctive pseudo-imperatives, noting that such compound expressions can be interpreted as a conditional imperative.

Many other details of Beardsley's account differ from what is proposed here. We simply note that basic theories of indicatives focus on the predicative content, rather than on any assertional relationship. In this paper the objective is to formalise a notion akin to predicative content for imperatives, not to deny the need for some understanding and formalisation of the relational notions of asserting and commanding. Whether or not refinement (Section 3.4) is the appropriate vehicle for formulating such reasoning in either case deserves further study.

6.5 Segerberg: a possible worlds account

Segerberg (1990) has an operator δ that takes a proposition p to the action that makes p true. The expression $!\delta p$ is then the *practical* formulae that making p true is commanded.

Actions are characterised extensionally: δp is modelled as a relationship between worlds. The intended intepretation is that it captures the possible outcomes of making p true in a given world. The formulae $[a]p$ means that p is always true following actions a.

As with other theories with explicit actions (van Eijck, 2000; Pérez-Ramírez & Fox, 2003a), actions can be combined by sequencing ($a_1; a_2$) and free choice ($a_1 + a_2$).

One constraint is that $[\delta p]p$ is true at all worlds in all models. Segerberg has it that δp is a subset of pairs of worlds in which the p holds in the second world. The aim is to avoid Ross' paradox, as this weakness means that disjunction introduction is not supported (in the notation of Chellas (1969, 1971) $!(a \vee b)$ does not follow from $!a$). This issue has been discussed at length in Section 3.2. Unfortunately, as Segerberg himself notes, and attempts to jusify, it also means that conjunction elimination is also not supported

$(!(a \wedge b)$ does not imply $!a)$.[48]

The resulting theory is rather too weak, as it fails to capture some uncontroversial, intuitively acceptable inferences. As with the theory proposed in this paper, it also does not consider quantification, anaphora, or the contextual updates required by some examples. This is observed by Lascarides & Asher (2004), who seek to build in this account in a way that allows more inferences while still avoiding the supposed problematic inferences of disjunction introduction.

6.6 Lascarides and Asher: a dynamic account

Next we consider the approach of Lascarides & Asher (2004), both because of its use of possible worlds, and because of its take on Ross' Paradox. They seek to address (i) Ross' observation on disjunction introduction with imperatives, and (ii) interaction between context and imperatives, in particular spatio-temporal implications in discourse, with the framework of Segmented Discourse Representation Theory (SDRT). As the authors indicate in the choice of examples, this latter issue is one that arises with both imperatives and indicatives, and their approach is to show that they have a uniform analysis of the phenomena with both indicatives and imperatives due to the neutrality of the relevant axioms of interpretation with respect to mood.

Given that it is not specifically an issue that arises exclusively with imperatives, it is beyond the scope of the objectives of the current paper, which merely seeks to formalise those issues that are of particular concern for imperatives.

In passing they remark on some examples of pseudo-imperatives. Although they agree with the view that conjunctive pseudo-imperatives with an "undesirable" proposition are not imperative, they claim that conjunctive pseudo-imperatives with a "desirable" proposition are imperative in nature, unlike Franke (2005a).

Lascarides & Asher note that in addition to being very weak, Segerberg's (1990) account (above, Section 6.5), does not capture quantification, anaphora, nor the contextual update required in sentences of the form

(105) *"Go to the traffic lights. There is a roundabout to your right."*

in which the second sentence be intepreted in a world in which the imperative has been complied with.

Like many before them, they reject the contention of Hare (1967), that the disjunction introduction rule should be valid, but only appears invalid because of Gricean scalar implicatures. This is because no details are given about how scalar implicatures would have the desired effect.[49] Instead, they

[48] Conceptually, this approach has some similiarities with Pérez-Ramírez & Fox (2003a) in distinguishing what is commanded — where we don't want to infer $p \wedge q$ from q — from the logical behaviour of the satisfaction criteria of what is commanded.

[49] As we have seen, in this paper we agree with Hare (1967), that the inference is valid, but that it seems invalid simply because of a confusion between *satisfaction conditions* and *refinement relationships*. Arguably, the notion of refinement has some connection with the Hare's notion of scalar implicatures.

argue that the inference is invalid, and that it should be blocked by requiring that the postcondition of the interpretation of δp (using the notation of Segerberg (1990)) should be $p \wedge \phi$, where ϕ is contextually determined.[50]

Lascarides & Asher define action expressions in terms of Discourse Representation Structures (DRSs) rather than propositions. In general, under the dynamic interpretation, DRSs transform the assignment of values to variables, and actions transform possible worlds. This approach is intended to deal conveniently with examples like (105). They note that the following interpretations are inadequate

(106) $$!\delta A \wedge B$$

(107) $$\delta A \wedge [\delta]B$$

the first because it does not formally update the context for the intepretatation of B, the second because they claim the derivation is impractical. It is worth noting the similarity between this example and conjunctive pseudo imperatives (Section 2.9). The difference is that there is no explicit conjunction or future tense in the propositional part. Apart from this, the analysis offered in this paper, following Franke (2005a), is similar to the first one here (except that formally we take the imperative to be an imperative, and not a practical formula). An account based upon this pseudo-imperative analysis does not then give rise to an appropriate ordering of events. We have adopted the same position as Lascarides & Asher, that this is a generic problem which is not specific to imperatives.

With the second proposal, (107) attributed to Seberberg, it could be argued that Lascarides & Asher are giving a way of producing an equivalent semantics. There are some subtle issue here. δA is interpreted as an imperative that is being complied with, not commanded, so there is a sense in which it is being viewed as a proposition, not an imperative as such. Our intepretation of a pseudo-imperative version of this example would contain the imperative itself, and the truth of the second conjunct would be conditional on the satisfaction of the imperative.[51] It is not immediately clear how the account would deal with the practical inferences such as (20), as such examples are not discussed. Presumably an account of such examples might require more consideration of the notion represented by expressions of the form $!\delta p$: that a particular action is commanded.

SDRS supports labelled discourse structures. Rhetorical relations are expressed in terms of these labels. These relations might indicate, for example, that the second DRS has to be interpreted in the context that results following the interpretation of the first DRS (as would be relevant in the above example), where the two DRSs in question interpret each of the constituent sentences. The relationship might also impose additional conditions, corresponding to ϕ above.

[50] It could be argued that this is very close to Segerberg (1990)'s account, except that the *relevant* subset of possible world pairs for an action δp is now specified, by way of ϕ, rather than being left open.

[51] Presumably Lascarides & Asher could deal with the conditional aspects of this example in the way by refining the axioms governing the rhetorical relations that they propose.

The analysis of the rhetorical relations is reductive in character. For example, a *Narration* relationship holds between two DRSs if the agents are in the same place at the end of the first and start of the second "events" described by the DRSs.

The argument is that (i) this contextual component (ϕ above) is a step towards addressing the weaknesses of Segerberg (1990) while still avoiding Ross' paradox, and (ii) the rhetorical relations (such as *narration*, *consequence* and *elaboration*) can be axiomatised to explain spatio-temporal implicatures.

While (ii) might seem a worthy, if challenging goal, some objections can be made concerning (i). Whether the supposed problem of Ross' paradox and the weaknesses of Segerberg (1990) are addressed is, in both cases, contingent on the nature of ϕ, rather than being intrinsic properties of the formalisation itself. Even if we were to agree that Ross' paradox is a problem, this does not seem entirely satisfactory. In this regard, it would again seem that these key issues are resolved by acknowledging a distinction between inferences involving the formal satisfaction criteria of imperatives, and those relationships involving the refinement of what is specified by an imperative in order to produce a plan of action. It would be interesting to see a reformulation of a possible worlds account that took such a distinction seriously.[52]

The account may provide insight into how best to deal with practical inferences (Section 3.1).

6.7 van Eijck: logics of action

The final system we consider because it presents a fairly comprehensive theory of some core issues using basic examples. On these grounds at least it is similiar to the approach proposed in the current paper.

van Eijck (2000) sets up a basic formal system of commands, modelled in terms of dynamic semantics, with actions denoting updates to states of affairs or worlds. There is no distinction between actual and intended change: it is assumed that commands are complied with. This is in contrast with the proposal of this paper, where there is a sense in which the satisfaction criteria given here correspond with a notion of "intended change" (Belnap & Perloff, 1992).

van Eijck initially concentrates on actions that have no repercussions. The basic actions/imperatives take the form of assignments of O or 1 to variables, with propositions being statements concerning the values of these propositional variables in a state. This is broadly in line with the model proposed here (Section A), except for the fact that commands are complied with immediately, rather than being "requests".

Commands are *valid* if they can always be performed. They are *satisfiable* if there are circumstances in which they can be performed. An action has another command as a consequence if it sets things up so that the second command can be performed (i.e. it is "satisfied"). van Eijck also mentions

[52] Lascarides & Asher (2004) go on to consider why imperatives given in answer to a question are not always commanded, and discusses a solution based upon narrative relations. From the current perspective, this is a pragmatic issue whose analysis needs to consider the semantics and pragmatics of questions and answers, and for these reasons is not considered here.

alternative notions of entailment between commands/imperatives: (i) where the consequent may be satisfied after the antecedent action; (ii) where the second action is also accomplished by (included in) the antecedent action.

van Eijck then goes on to consider a formalisation in terms of actions with repercussions, where there is interference between the values of variables that can be changed due to causation, modelled by accessibility relations. An action is intended to be performed with minimal impact on the state. This allows an agent's skill to be modelled, together with a notion of intrinsic change that should allow von Wright's (1983) notions of **make** p, **keep** p, **end** p, and **prevent** p to be modelled.

The account given in the current paper side-steps some of these concerns about modelling causation, and the varieties of actions. Although these would be needed to give a reductive account of such actions, such an analysis is independent of how we deal with imperatives: it is possible to model the general logical behaviour of imperatives without giving an account of varieties of actions, and it is possible to give an account of "make", "keep", "end" and "prevent" etc. independently of any imperative phenomena (the issue of their meaning arises with simple indicatives; they are not a specific concern of imperatives).

The simplicity of the actions in this account, and the fact that imperatives are assumed always to be complied with means that some issues are not so clear. But as with many related approaches, we see that there is an intimate relationship between an action (or the formalisation of an action) and its postconditions, so much so that the action could be abstracted away, much as proposed in the current paper.

Assuming that disjunction ($\vee$) were given its usual interpretation, it would seem that the initial formalisation at least supports the introduction of disjunction.[53]

6.8 Franke: pseudo imperatives

The account of pseudo imperatives given in Section 2.8 and 2.9 broadly adopts the proposal of Franke (2005b). This is not to say that this is the only available account of such phenomena, or that its analysis is beyond question. The objective here has been largely to illustrate how it is possible to develop a theory with sufficient flexibility to allow the interpretation of expressions as imperatives or propositions as required, rather than being forced into a particular view by the machinery of the logic.

6.9 Possible Worlds: some general comments

Possible worlds is a commonly assumed framework, both in the semantics of natural language and specifically in the formal treatment of imperatives (for example, Kamp (1979), Segerberg (1990) (Section 6.5), van Eijck (2000) (Section 6.7), Lascarides & Asher (2004) (Section 6.6) and Franke (2005b)) (Section 6.8). Typically, imperatives are seen as being satisfied by actions

[53] That is, $[\pi]p \models [\pi](p \vee q)$ in the notation of van Eijck's paper.

which are modelled by an accessibility relation; performing an action results in a change to the current world.

Although such a framework may provide a useful model for imperatives and satisfaction, they may require us to assume too much for our purposes, for example about the nature of time, actions, and causation. Also, although actions may be taken to satisfy an imperative, possible worlds leaves them outside the realm of discussion, as they are not part of the state, so we can only talk about them indirectly in terms of their extensional impact (as characterised by sets of possible worlds).

Instead of selecting such a framework as a given, without assessing its merits, and then being obliged to find ways of mapping the phenomena in question into the framework, it seems more appropriate first to formalise the desired patterns of behaviour directly. This avoids the need to work around the presumptions and artifacts of a framework which was not conceived with imperatives in mind, and for which there is no guarantee that it is at an appropriate level of abstraction to capture all the salient details in a comprehensible fashion.[54]

Once we have proof rules that capture the intuitively acceptable patterns of behaviour (at least to a first approximation), we can then formally assess how well an interpretation in a given model, such as a possible worlds model, mirrors our understanding as expressed in the proof rules, and whether such a model captures our intentions *appropriately*. This approach avoids making the preliminary formalisation overly dependent on tricky notions that are hard to analyse correctly.

7 Conclusions

This paper explores a proof-theoretic treatment of satisfaction conditions for imperatives. It has sought to abstract away from problematic notions, such as actions and events. The focus has been on core, or paradigm examples, with extended discussion on the nature of the patterns of inference that should be supported by a satisfaction relationship. This is viewed as corresponding to the standard notion of entailment with indicatives.

Phenomena that are not exclusive to imperatives have not been considered. This includes context change, anaphora, and quantification.[55] There does not appear to be overwhelming evidence that imperative-specific accounts of such issues are required. Aspects concerning the pragmatic intepretation of imperatives have not been addressed by this semantic account, neither have background prohibitions and, for example, the defeasibility of Free Choice disjunction been considered (Asher & Bonevac, 2005). In the latter case at least, this is because such defeasible inference are not exclusive to

[54]Given that a typical possible worlds framework has access to the full machinery of set-theory, it should be possible to find *some* way of capturing the behaviour of imperatives, but it is not clear what explanatory power this would have. See Fox (2000) for a more general discussion of this point.

[55]It is by no means certain that the analysis of quantifaction of with imperatives is straightforward. Consider *"Everybody dance now"* (Mastop, 2005).

imperatives; indicatives may also require some form of non-monotonicty in order to cope with new information that conflicts with earlier information.

Even so, in the absence of an account of such phenomena, the analysis might seem rather weak in its coverage of linguistic data. This is exacerbated by the absence of any account of an appropriate syntax-semantics interface. However, the key objective of the paper has been to formulate a logic of satisfaction (Hare, 1967) for imperatives, broadly supporting the notions of fulfillment and outcomes proposed by Lappin (1982) and Ginzburg & Sag (2000), and to come to a deeper understanding of what inference means in the context of a logic of satisfaction.

Some of the inference rules of the proposed logic of satisfaction may appear to be counter-intuitive. This is of particular concern with inferences governing disjunction, where it has been argued that the inference from a to $a \vee b$ should actually run in the other direction (Kenny, 1966; Kamp, 1973). This paper takes the view that this perceived problem arises due to confusion between *inferences* involving satisfaction conditions with a notion of *refinement*. The "inference" from $a \vee b$ to a (or b) is really saying that one way of achieving $a \vee b$ is for a (or b) to be achieved.[56] This is refinement (or *reification*) in the sense used in computer science (Wirth, 1971). A parallel may be drawn between practical inference with imperatives and the process of executing a `Prolog` program in order to satisfy a set of propositional constraints.

Although the current paper focuses on the satisfaction inferences, any comprehensive semantic account of imperatives should also consider these patterns of refinement, and the notion of "practical inference" and "validity"[57] in which they appear to be rooted. In the case of the current theory, there is a need to reconcile the proposed approach to practical inference (Section 3.1 with the treatment of pseudo-imperatives (Sections 2.8 and 2.9).

There are further issues surrounding the presuppositions of imperatives. The current paper makes some tentative suggestions about this, but some additional reflection is needed to determine whether this appropriately captures the idea that uttering an imperative gives rise to the perception of possible courses of action. Other issues that require consideration include the relationship with deontic modality (permissions and obligations). It is intended that these and related topics will be the subject of future work.

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[56] And further, that achieving $a \vee b$ does not guarantee that we achieve a.

[57] In the sense of Ross (1941), where we attempt to reason with what is understood to have been commanded, as suggested Beardsley (1944).

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A Sketch of a Model

Here we sketch a model that can interpret the core theory, with conditional and pseudo imperatives.

(108) **Basic Notions.** Both agentive properties and propositions are interpreted as expressions in some language L with appropriate closure rules. We then define functions $(\mathsf{T}_s, \hat{\mathsf{T}}_s^{\alpha}, \hat{\mathsf{T}}_s^{!\alpha})$ of type $\mathsf{L} \longrightarrow \mathsf{2}$ that evaluate, in a state s, the interpretations propositions (T_s), and agentive properties in propositional $(\hat{\mathsf{T}}_s^{\alpha})$ and imperative $(\hat{\mathsf{T}}_s^{!\alpha})$ contexts with respect to agent α.

Some coherent model of tense is assumed, where there is an accessibility relation between states of an appropriate for to evaluate expressions in the context of future states, and sustain any appropriate inference rules and axioms.

(109) **Agentive Properties**

The class of expressions AP that interpret agentive properties are generated from atomic agentive properties $\mathsf{AP}_{\text{atomic}} \subseteq \mathsf{AP}$ using the following rules

(a) If $[\![p]\!] \in \mathsf{AP}_{\text{atomic}}$ then $[\![p]\!] \in \mathsf{AP}$

(b) If $[\![p]\!] \in \mathsf{AP}$ then $[\![- p]\!] \in \mathsf{AP}$

(c) If $[\![p]\!], [\![p']\!] \in \mathsf{AP}$ then $[\![p \cap p']\!] \in \mathsf{AP}$

(d) If $[\![p]\!], [\![p']\!] \in \mathsf{AP}$ then $[\![p \cup p']\!] \in \mathsf{AP}$

(110) **Imperatives.**

The expressions I that interpret imperatives is closed under the following rules.

(a) If $[\![p]\!] \in \mathsf{AP}, [\![\alpha]\!] \in \mathsf{A}$ then $[\![p!_{\alpha}]\!] \in \mathsf{I}$

(b) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![i \rightarrow \phi]\!] \in \mathsf{I}$

(c) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![\phi \rightarrow i]\!] \in \mathsf{I}$

(d) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![i \vee \phi]\!] \in \mathsf{I}$

(111) **Propositions.**

The expressions P that interpret propositions is closed under the following rules.

(a) If $[\![p]\!] \in \mathsf{AP}, [\![\alpha]\!] \in \mathsf{A}$ then $[\![p(\alpha)]\!] \in \mathsf{P}$
(b) If $[\![\phi]\!], [\![\phi']\!] \in \mathsf{P}$ then $[\![\phi \wedge \phi']\!] \in \mathsf{P}$
(c) If $[\![\phi]\!], [\![\phi']\!] \in \mathsf{P}$ then $[\![\phi \vee \phi']\!] \in \mathsf{P}$
(d) If $[\![\phi]\!], [\![\phi']\!] \in \mathsf{P}$ then $[\![\phi \rightarrow \phi']\!] \in \mathsf{P}$
(e) If $[\![\phi]\!] \in \mathsf{P}$ then $[\![\neg\, \phi]\!] \in \mathsf{P}$
(f) If $[\![\phi]\!] \in \mathsf{P}$ then $[\![\diamond \phi]\!] \in \mathsf{P}$
(g) If $[\![i]\!] \in \mathsf{I}$ then $[\![\blacklozenge\, i]\!] \in \mathsf{P}$
(h) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![i \rightarrow \phi]\!] \in \mathsf{P}$
(i) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![\phi \rightarrow i]\!] \in \mathsf{P}$
(j) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![i \vee \phi]\!] \in \mathsf{P}$
(k) If $[\![i]\!] \in \mathsf{I}, [\![\phi]\!] \in \mathsf{P}$ then $[\![i \wedge \phi]\!] \in \mathsf{P}$

(112) **Evaluation of propositions.**

The function T_s evaluates the truth of the interpretation of a proposition.

If $[\![\phi]\!], [\![\phi']\!] \in \mathsf{P}$, then

(a) $\mathsf{T}_s[\![\phi \wedge \phi']\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ and $\mathsf{T}_s[\![\phi']\!] = 1$,
(b) $\mathsf{T}_s[\![\phi \vee \phi']\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ or $\mathsf{T}_s[\![\phi']\!] = 1$,
(c) $\mathsf{T}_s[\![\phi \rightarrow \phi']\!] = 1$ iff $\mathsf{T}_s[\![\phi']\!] = 1$ whenever $\mathsf{T}_s[\![\phi]\!] = 1$,
(d) $\mathsf{T}_s[\![\neg\, \phi]\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 0$,
(e) $\mathsf{T}_s[\![\diamond \phi]\!] = 1$ iff $\mathsf{T}_{s'}[\![\phi]\!] = 1$ for some s' in the future wrt s.

Otherwise the left-hand expression evaluates to 0 in all cases.

(113) **Propositional evaluation of agentive properties.**

The function $\hat{\mathsf{T}}^{\alpha}_{s}$ evaluates the truth of the interpretation of an agentive property in a state s when applied to an agent α to form a proposition.

If $[\![p]\!], [\![p']\!] \in \mathsf{AP}$ and $[\![\alpha]\!] \in \mathsf{A}$, then

(a) If $[\![p]\!] \notin \mathsf{AP}_{\text{atomic}}$ then $\mathsf{T}_s[\![p(\alpha)]\!] = \hat{\mathsf{T}}^{\alpha}_{s}[\![p]\!]$
(b) $\hat{\mathsf{T}}^{\alpha}_{s}[\![p \cap p']\!] = 1$ iff $\hat{\mathsf{T}}^{\alpha}_{s}[\![p]\!] = 1$ and $\hat{\mathsf{T}}^{\alpha}_{s}[\![p']\!] = 1$,
(c) $\hat{\mathsf{T}}^{\alpha}_{s}[\![p \cup p']\!] = 1$ iff $\hat{\mathsf{T}}^{\alpha}_{s}[\![p]\!] = 1$ or $\hat{\mathsf{T}}^{\alpha}_{s}[\![p']\!] = 1$,
(d) $\hat{\mathsf{T}}^{\alpha}_{s}[\![-\, p]\!] = 1$ iff $\hat{\mathsf{T}}^{\alpha}_{s}[\![p]\!] = 0$,
(e) If $[\![p]\!] \in \mathsf{AP}_{\text{atomic}}$ then $\hat{\mathsf{T}}^{\alpha}_{s}[\![p]\!] = \mathsf{T}_s[\![p(\alpha)]\!]$

In cases (113b)–(113d), the left-hand expression otherwise evaluates to 0.

(114) **Imperative evaluation of agentive properties.**

The function $\hat{\mathsf{T}}^{!\alpha}_{s}$ evaluates the satisfaction of the interpretation of an agentive property in a state s when used to form an imperative addressed to agent α.

If $[\![p]\!], [\![p']\!] \in \mathsf{AP}$ and $[\![\alpha]\!] \in \mathsf{A}$, then

(a) $\mathsf{T}_s[\![\blacklozenge\, p!_\alpha]\!] = \hat{\mathsf{T}}^{!\alpha}_s[\![p]\!]$

(b) $\hat{\mathsf{T}}^{!\alpha}_s[\![p \cap p']\!] = 1$ iff $\hat{\mathsf{T}}^{!\alpha}_s[\![p]\!] = 1$ and $\hat{\mathsf{T}}^{!\alpha}_s[\![p']\!] = 1$,

(c) $\hat{\mathsf{T}}^{!\alpha}_s[\![p \cup p']\!] = 1$ iff $\hat{\mathsf{T}}^{!\alpha}_s[\![p]\!] = 1$ or $\hat{\mathsf{T}}^{!\alpha}_s[\![p']\!] = 1$,

(d) $\hat{\mathsf{T}}^{!\alpha}_s[\![- p]\!] = 1$ iff $\hat{\mathsf{T}}^{!\alpha}_s[\![p]\!] = 0$,

(e) If $[\![p]\!] \in \mathsf{AP}_{\text{atomic}}$ then $\hat{\mathsf{T}}^{!\alpha}_s[\![p]\!] = 1$ iff $\mathsf{T}_s[\![\diamond p(\alpha)]\!] = 1$.

In cases (114b)–(114e), the left-hand expression otherwise evaluates to 0.

(115) **Propositional evaluation of hybrid expressions.**

If $[\![i]\!] \in \mathsf{I}$ and $[\![\phi]\!] \in \mathsf{P}$, then

(a) $\mathsf{T}_s[\![p \rightarrow i]\!] = 1$ iff $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$ whenever $\mathsf{T}_s[\![p]\!] = 1$,

(b) $\mathsf{T}_s[\![i \rightarrow \phi]\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ whenever $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$,

(c) $\mathsf{T}_s[\![i \vee \phi]\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ whenever $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 0$,

(d) $\mathsf{T}_s[\![i \wedge \phi]\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ whenever $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$.

Otherwise the left-hand expression evaluates to 0 in all cases.

(116) **Imperative evaluation of hybrid expressions.**

If $[\![i]\!] \in \mathsf{I}$ and $[\![\phi]\!] \in \mathsf{P}$, then

(a) $\mathsf{T}_s[\![\blacklozenge\, p \rightarrow i]\!] = 1$ iff $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$ whenever $\mathsf{T}_s[\![p]\!] = 1$,

(b) $\mathsf{T}_s[\![\blacklozenge\, i \rightarrow \phi]\!] = 1$ iff $\mathsf{T}_s[\![\phi]\!] = 1$ whenever $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$,

(c) $\mathsf{T}_s[\![\blacklozenge\, i \vee \phi]\!] = 1$ iff $\mathsf{T}_s[\![\blacklozenge\, i]\!] = 1$.

Otherwise the left-hand expression evaluates to 0 in all cases.

(117) **Model of judgements.**

(a) $M \models \phi : \mathsf{Prop}$ iff $[\![\phi]\!] \in \mathsf{P}$

(b) $M \models i : \mathsf{Imp}$ iff $[\![i]\!] \in \mathsf{I}$

(c) $M \models \phi\ \ \mathsf{True}$ iff $\mathsf{T}_\sigma[\![\phi]\!] = 1$ where σ is the "current" state.

(118) **Claim (soundness)** If B follows from A using the proof theory of Section 2, then for all M such that $M \models A$, it follows that $M \models B$.

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