Imperatives as obligatory and permitted actions

Miguel Pérez-Ramírez
(mperez@essex.ac.uk *and* mzerep2000@yahoo.com)
Chris Fox
(foxcj@essex.ac.uk)
Computer Science Department
University of Essex
Wivenhoe Park, Colchester CO4 3SQ,
United Kingdom

Abstract

We present a dynamic deontic model for the interpretation of imperative sentences in terms of Obligation (O) and Permission (P). Under the view that imperatives prescribe actions and unlike the so-called "standard solution" (Huntley, 1984) these operators act over actions rather that over statements. Then by distinguishing obligatory from non obligatory actions we tackle the paradox of Free Choice Permission (FCP).

1. Introduction

The aim of this paper is to provide a model for the interpretation of direct imperative sentences in terms of Obligation and Permission. The model possesses properties that corresponds with our intuitions about the use of obligatory imperatives and is not affected by inferential problems such as the paradox of Free Choice Permission (von Wright, 1968;1991;1999; Wieringa and Meyer, 1993, Kamp, 1973), which in non-deontic approaches is known as Ross's counterexample (Ross, 941; von Wright, 1968).

In general imperatives are conceived as sentences used to issue orders or commands (Radford 1997, Lyons, 1968; Nodine, 1996; Megginson, 1996; MacFadyen, 1996). Sentences involving requests, threats, exhortations, permissions, concessions, warnings, advices, etc. are also included as imperatives (see Huntley, 1984; Sperber and Wilson, 1986; Hamblin, 1986 for an extensive classification of imperatives). This is a broad characterisation, which may include other types of sentences. For simplicity of exposition, we will adopt a syntactic view of *direct imperatives*, since this view locates appropriate sentences[1] in a language and allows us intuitively to distinguish them from statements and questions. We shall adopt the following definition.

Definition: *Imperatives are sentences used to ask someone to do or not to do something and that do not denote truth-values.*

This 'something' we shall call *requirement*. Examples are *Come here*! and *Stop*! They have no truth conditions as such.

Jorgensen (1937) began the debate on the modelling of imperatives. His main concern was to account for the inferential role of imperatives under apparent contradiction that they cannot be *true* or *false*. Having the imperative a) *Love your neighbours as yourself*! and the premise b) *Alison is your neighbour* it seems reasonable to infer c) *Love Alison as yourself* even thought a) and c) cannot be *true* or *false* (this pattern is called Jorgensen's dilemma, see Ross, 1941; Walter, 1996). Since 1937 there have been different ways of approaching imperatives (Weinberger, 1991). In particular the *standard solution* (Huntley, 1984) describes a common features in different approaches. Its key characteristic is the assumption that the core meaning of imperatives is propositional: something that can be *true* or *false*. Typical of this approach is to split imperatives into two parts: a descriptive or propositional and a prescriptive part[2]. Jorgensen's offers a typical approach akin to the 'standard solution' where, for instance, *Post the letter*! is translated into !*The letter has to be posted*, using the symbol '!'

[1] Imperatives are a type of sentence. Levinson says "it seems that the three basic sentence types, *interrogative*, *imperative*, and *declarative* are universals, all languages appear to have at least two and mostly three of these" Levinson (1981: p. 242).

[2] Hare (1952) used the terms *phrastic* and *neustic* respectively to refers these two components

and a statement. In this way classical logic is used to account for practical inference[3] leading to the derivation of unintuitive conclusions (see description in Pérez-Ramírez, 2000; 2002).

Deontic logic focuses on imperatives by using deontic concepts such as *obligation* and *permission* (von Wright, 1968: p. 14). It is one of the most prolific branches devoted to the study of imperatives (norms: sentences conveying obligation or permission) by using the operators Op and Pp, where usually p is a statement.

von Wright (1968) presents a standard deontic logic which is akin to the standard solution. This logic validates the counterintuitive expression $Op \rightarrow O(p \vee q)$ which means that if 'it is obligatory that p' then 'it is obligatory that $p \vee q$'. For instance, "If one ought to mail a letter, one also ought either to mail or to burn it" (von Wright, 1968: p. 20). If $O(p \vee q)$ means that it is obligatory that $p \vee q$, a hearer may choose to perform q.

In more recent approaches the operator P acts over actions. However Dignum et al. (1996) say that event thought dynamic logic solve some paradoxes in standard deontic logic, the paradox of FCP still remains under the form $P(\alpha) \rightarrow P(\alpha+\beta)$ where α and β are actions and + is the choice operator. They illustrate with the example P(*Talk to the president*) $\rightarrow$ P(*Talk to the president* + *Shoot the president*).

They propose a logic in which P acts over actions and they distinguish strong (P_s) and weak permission (P_w). According to them these operators when applied to statements satisfy $P_w(p \vee q) \equiv P_w p \vee P_w q$ and $P_s(p \vee q) \equiv P_s p \wedge P_s q$. They face new problems, for instance their logic contains the expression $P_s(\alpha) \rightarrow P_s(\alpha \& \beta)$. They explain, "if α is permitted, it is (also) permitted in any combination with other actions". So the logic validates the example P(*fire a gun*) $\rightarrow$ P(*fire a gun* & *aim at the president*). They propose a solution to these new problems by making reference to context.

There have been different proposals to solve the paradox of FCP but according to von Wright, even though a huge literature on the topic has grown up, a universally accepted solution to these difficulties has not yet been found (von Wright, 1999).

The paradox of FCP is an inferential problem initially caused by the scope of the classical logic connectives when used with non-propositional objects. Given a statement we can introduce any other statement to form a disjunction. Also given a conjunction of statements we can eliminate one of the components. These properties are not desirable when modelling imperatives. On the other hand, approaches in which O and P operate over actions try to restrict the scope of dynamic operators by introducing new operators and sometimes also new problems. Thus, either we should restrict the application of classical logic rules when operating on statements in the scope of O and P, or else we can consider O and P to be applied to something other than statements, and consider the rules that are appropriate for the operations on such expressions. It is the latter option that we take here.

In this paper we follow the intuitions that imperatives prescribe actions. Thus, a dynamic deontic logic (L_{DL}) is developed here in which the operators for Obligation O(-) and permission P(-) operate over actions rather that over statements. The intended meaning of the operators O(-) is "*It is obligatory to* –" and for P(-) is "*It is permitted to* –" where "–" is the place for the requested action. The multimodal operator '[-]' is used to model actions behaviour (Harel, 1979) and Hoare's triple ($Pre \rightarrow [\alpha] Pos$ where α is an action, *Pre* are pre-conditions and *Pos* are post-condition) is used to verify correctness of actions. We use the dynamic operators composition ';' and choice '+' to model conjunction and disjunction of requirement respectively. It is also used the intuition of encapsulating what is obligatory in order to distinguish what is obligatory from what is not.

It will be shown that the distinction between obligatory actions and simple actions solves the problem of FCP and the model behaves according to our intuitions about the use of imperatives.

In particular, if an action is obligatory two things are assumed a) it belongs to a set where all the obligatory actions are kept and b) it is satisfiable (the action can be performed). The second assumption is analogous to Chellas's axiom where 'ought' implies 'can' (Chellas, 1971: p. 125).

The remainder of the paper is organised as follows: Section 2 analyses some properties of imperatives and their relation with the paradox of FCP so that we know what we need to model. Section 3 presents the model that involves the main features observed in Section 2. Section 4 illustrates the use of the model to solve some problems. Section 5 includes the main conclusions of this paper and it is followed by a list of references.

---

[3] The term *practical inference* usually refers inferences in which imperatives take part as premises

2. Analysis

Before providing a model for the inferential behaviour of obligatory imperatives. Here we will emphasise that the use of classical logical connectives between imperatives is not appropriate, and that as imperatives prescribe actions, such actions, and the operations between them, (as modelled by Dynamic Logic) are a more appropriate basis for modelling imperatives. Thus, we will also argue that obligation and permission (O and P) should operate over imperatives, not statements.

The view of imperatives in terms of action immediately solves some counterintuitive results such as the lack of truth–values. Then it is crucial the observation that what is obligatory should be kept encapsulated in order to solve the paradox of FCP in a simple way that also corresponds with what people do when interpreting imperatives.

2.1 Classical logic connectives and imperatives

Intuitively, orders appears to involve the use of logic connectives (*and, or, not, if-then-else*). For example *do X and Y*, *do X or Y*, *Don't do X*, *If C then Do X*, etc., seem to be appropriate constructs. Other connectives (*while, after, before, until* etc.) seem also to yield appropriate constructs. For instance, *while C do X, do X until C, do X before C and do X after C.* (where *X*, *Y* are requirements, *C* represents conditions and 'do' and 'don't' are used here only to indicate that *X*'s and *Y*'s are requests but do not make reference to operators).

The use of the words (*and, or, not, if-then-else*) in imperatives and their similarity with classical logic operators has perhaps led to the assumption that the classical operators are appropriate to model the imperative connectives. Alchurrón and Martino (1990: p. 56), referring to the classical connectives 'and,' 'no' and 'or', say that when looked at closely, the behaviour of these connectives between norms is not unlike their behaviour between statements and more generally in all propositional contexts.

To this respect Von Wright says, "Thus, if we were to build a formal logic of imperatives we could, without creating ambiguity and confusion, apply sentential connectives to imperatives sentences." (von Wright, 1991). For him the imperative is viewed as O*p* which reads "it is obligatory that *p*". However in his analysis he still uses classical logic connectives within the scope of P and O.

We must not be misled for the apparent similarity without analysing whether or not classical connectives have the appropriate behaviour.

It does seem that classical logic is suitable to account for imperative satisfaction (Ross, 1941). *Close the door and sit down!* is satisfied if it is the case that *The door is closed* ∧ *The hearer has sat down*. *Shut up or get out of here*! is satisfied when it is the case that *Your are silent* ∨ *you are not here*. *Don't turn the lights off!* is satisfied when it is the case that *The lights remain on*. This provides evidence of the role that classical logic can play in the analysis of imperatives.

Note that there are aspects of imperatives that clash with the assumption mentioned above. We illustrate here with examples the inappropriateness of the classical connectives between imperatives. *Come here!* might not be obligatory when this order is given by a soldier to a general, because of the lack of authority. If the soldier instead says to the general *Don't come here!*, the negation of the first command, it is still not obligatory because again the soldier does not have the necessary authority. Obligation of imperative negation does not follow classical negation. FCP provides evidence of the wrong conclusions derived from the use of the rule of introduction for disjunction. Finally some imperatives might involve a physical dependence when it is not possible to satisfy one without satisfying a previous one. For example, *Write a letter and send it to your family!* might be obligatory in some contexts but not *Send the letter to your family and write it!* Dependence determines a serial satisfaction of the imperative but most importantly, it indicates that *commutativity* is not a property always shared by imperative conjunctions (or rather sequences). Thus the basic classical operators (negation, disjunction and conjunction) all present some property that is not followed by obligatory imperatives and imperatives in general. Imperative properties clash with classical logic.

2.2 Physical action and imperatives

We have said that imperatives convey requirements. Requirements are requests for action or prescriptions of actions. This view is shared by other authors (Ross (1941), von Wright (1968), Hamblin (1987: p. 45) and

Segerberg (1990) among others). For instance for Segerberg (1990), imperatives are prescriptions for actions. For Ross (1941: p. 54), an imperative is a sentence that expresses an immediate demand for action but that does not describe a fact. For him an imperatives is satisfied if we have the result of an agent's action.

Referring the normative concepts obligation, permission and prohibition, von Wright says, "Normative concepts are used in two principal ways. They are used prescriptively in normative discourse for enunciating rules of action and other norms, e.g. for giving permission, imposing an obligation, or granting a right. But they are also used in descriptive discourse for speaking about norms, e.g. for saying that according to a certain code a certain action is forbidden" von Wright (1968: p. 11).

Radically Hamblin suggests that the core meaning of an imperative is an action. "We can analyse, as it were, the kernel or content of the imperative – **the action** (though the word is not sufficiently general) that the imperative enjoins- without worrying about the way in which it enjoins it …" Hamblin(1987: p. 45, emphasis is the author's).

An imperative might be used to indirectly change the world through the hearer or to prevent an action, by conveying a requirement. Some examples of prescriptions of actions are, *Write a letter!* that prescribes the action of writing and *Come here!* that prescribes the action of approaching. Examples of imperatives requiring that a state of affairs remains unchanged by forbidding or preventing actions are *Don't close the door!* and *Don't turn the lights off!* Thus prescribing and forbidding actions are included in our definition of imperative and requirements. In our model, imperatives will prescribe obligatory or permitted actions. The following expressions illustrate schemas of obligatory imperatives in which dynamic operators model the operators between requirements.

- **O(**a**)** — ***It is obligatory to*** a
- **O(**$\alpha_1 ; \alpha_2$**)** — ***It is obligatory to*** α_1 AND α_2
- **O(**$\alpha_1 + \alpha_2$**)** — ***It is obligatory to*** α_1 AND α_2
- **O(**$\phi \Rrightarrow \alpha$**)** — ***It is obligatory to*** α_1 AND α_2
- **O(**$(\phi \Rrightarrow \alpha_1)+(\neg\phi \Rrightarrow \alpha_2)$**)** — ***It is obligatory to*** (α_1 if ϕ else α_2)

An action is not *true* or *false*. The operator ';' describes a sequencing suitable to model the property of dependence and therefore the non commutativity of a conjunction of actions. We will see that there is not a rule to eliminate an action from a conjunction of actions. The operator '+' describes a disjunction of actions.

2.3 Obligation and permission

One of the first intuitions about obligatory and permitted actions is that if the action is obligatory, then it is permitted. That is, if we assume a fix state of affairs, the set of all obligatory actions in that state is contained in the set of permitted actions in that state. See Fig. 1 below, where *PAct* is the set of all permitted actions, *OAct* is the set of all obligatory actions, and *CAct* is the complement set between permitted and obligatory actions.

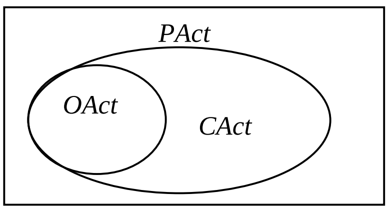

Figure 1: *Relation between Obligatory (OAct) and Permitted (PAct) actions*

Usually it is assumed that O and P are inter-definable and can be substituted each other under the assumption that $O(\alpha) = \neg P(\alpha')$ and $P(\alpha) = \neg O(\alpha')$, where α is an action and α' is the negation of the action α. That is, if an action α produces the result Q, then α' produces the result $\neg Q$. For the reader akin to this terminology, we can define the negation of composed actions as follows.

- If a is an atomic action then a' is the negation of a
- The negation of $(\alpha_1 ; \alpha_2)$ is $(\alpha'_1 + \alpha'_2)$
- The negation of $(\alpha_1 + \alpha_2)$ is $(\alpha'_1 ; \alpha'_2)$
- The negation of $(\phi \Rrightarrow \alpha)$ is $((\neg\phi)? + \alpha')$

Example: α=*Come here* α'=*Don't* (*Come here*)

Negation of composed actions does not necessarily correspond with common use of sentences in language but logically actions behave according to the formulation above.

von Wright seems to hesitate about the idea of considering that O and P are interdefinable (von Wright, 1991: p. 17). Weinberger argues, "If we understand normative systems as control systems as control systems we cannot chose permission as the basic operator, because a system embracing only permission cannot function as a control system, since permission does not exclude any possible state" (Weinberger, 1991: p. 289).

2.4 Communicative action

In recent approaches for the interpretation of the utterances the concept of *communicative action* has become increasingly popular. As here context is thought of a set of beliefs, then *communicative action* is described in terms of context changes. Perhaps more important is that communicative action is used to describe the meaning of utterances. That is, meaning of utterances is described in terms of context changes.

For instance, in his dynamic interpretation analysis Bunt (2000) says "Communicative action, as opposed to physical action, cannot change anything in the physical world, but only something in the 'mental worlds' of the communicating agents, …" (Bunt, 2000: p. 97). The term 'mental' refers to the conception of context as a set of beliefs.

This concept corresponds with our intuitions about interpretation of utterances but we also should note that context is conceived as 'a set' of beliefs. In a set we might apply operators among sets such as verifying membership, but not inferring membership to the set by applying logical rules. For instance if we believe that we are under the obligation to *Post a letter* we do not believe that we are obliged to *Post the letter or to burn the letter*. That is if P is in our set of beliefs, it does not mean that $P \vee Q$ is in such set.

Applying this concept to the interpretation of imperatives, if we are ordered to do α and if we believe that it is obligatory to do α, this does not mean that we believe that it is obligatory to do $\alpha+\beta$. Our beliefs are encapsulated and the membership of a belief in our set of beliefs and its updating process is not governed only by classical logic rules.

2.5 Obligation in a context

Intuitively *context* refers to the situational information or state of affairs where imperatives and other type of sentences are uttered. Context might involve a variety of different factors such as situation, agents (speaker and hearer) and their roles, among others, however it plays a crucial role in the interpretation of imperatives and that depending on the conditions provided by a context, an imperative can be obligatory in that context. For instance in a military environment a soldier is not usually in position to give orders to a general.

Authors such as Sperber and Wilson (1986), Bunt (2000), Manara and De Roeck (1997) agree that context is related to people's view or perception of the world or a particular situation rather than the world or the situation themselves. These authors conceive context in terms of what people have in their minds. That is, the concept of context includes the broad band of beliefs, knowledge and intentions. All of them can be represented as propositions. Following these authors we subscribe to the view that:

Definition: *A context is a consistent*[4] *collection of propositions that reflects a relevant subset of agents' beliefs.*

Nevertheless, the view of context as a set of propositions (see Buvac, 1995) will not commit us here to an ontology or classification of components or to the use of operators such as **B** for beliefs and **K** for knowledge (see Ramsay, 2000; Turner, 1992). We simply assume that all that which constitutes a context can be represented in terms of propositions so the context is viewed as a consistent set of propositions and with Chellas (1971) we agree that an imperative can be obligatory in some contexts but not in others.

[4] A proposition is consistent in a context if there is no contradiction between that proposition and the set of propositions representing the context.

3. Model for imperatives in terms of Obligation and Permission

The model we present here goes along the lines of First-order dynamic logic, similar to the logic provided by Harel (1979), in which the operators for obligation and permission are introduced in the language L_{DL}. Then we provided a possible worlds semantics. Once the set of action is defined we encapsulate the actions that are obligatory or permitted. Thus the obligation or permission of actions is given in terms of the membership of the actions in turn in these sets.

3.1 Syntax of L_{DL}

3.1.1 Definition of sets

We assume the following sets.

C	$= \{c, c_1, c_2, \ldots\}$	is a set of constant symbols
V	$= \{x, y, z, \ldots\}$	is a set of variable symbols
F	$= \{f, g, h, \ldots\}$	is a set of function symbols
$AtAct$	$= \{a, a_1, a_2, \ldots\}$	is a set of atomic actions
$AtPred$	$= \{p, q, r, \ldots\}$	is a set of atomic predicate symbols

3.1.2 Definition of terms

$$t = c|v|f(t_1, t_2, \ldots, t_n)$$

The recursive definition of terms is as follows: a term is a constant (c), a regular variable (v), or a function ($f(t_1, t_2, \ldots, t_n)$) of arity n (n arguments), where f is a function and $t_1, t_2, \ldots, t_n$ are terms.

3.1.3 Syntax of L_{DL}

It is assumed here that *Pred* contains all formulae of L_{DL}. Thus, if $p \in AtPred$, $t_1, t_2, \ldots, t_n$ are terms, $x \in V$, and $\alpha \in Act$, then all possible wffs in L_{DL} are defined as follows:

$$\boldsymbol{\phi = | p(t_1, t_2, \ldots, t_n) | t_1 = t_2 | \neg\phi | \phi_1 \wedge \phi_2 | \exists x\phi | O(\alpha) | P(\alpha) | [\alpha]\phi}$$

where $p(t_1, t_2, .., t_n)$ is an atomic formula (predicate), with arity n. $t_1 = t_2$ is the equality predicate (=). $\neg\phi$ is the negation of ϕ. $\phi_1 \wedge \phi_2$ is the conjunction of formulae ϕ and ψ. $\exists x\phi$ is the existential quantifier. O(α) indicates that α is obligatory. P(α) indicates that α is permitted. $[\alpha]\phi$ is a modal expression indicating that ϕ holds after the action α is performed.

The usual abbreviations are used:

- $\phi_1 \vee \phi_2 \ = \ \neg(\neg\phi_1 \wedge \neg\phi_2)$,
- $\phi_1 \rightarrow \phi_2 \ = \ \neg\phi_1 \vee \phi_2$
- $\phi_1 \leftrightarrow \phi_2 \ = \ \neg\phi_1 \rightarrow \phi_2 \wedge \phi_2 \rightarrow \phi_1$
- $\forall x\phi \ = \ \neg\exists x\neg\phi$
- $[\alpha]\phi \ = \ \neg\langle\alpha\rangle\neg\phi$

Note that both O and P are introduced in L_{DL} and both of them and both operate over actions. If α is an action, O(α) stands for *it is obligatory to* α and P(α) stands for *it is permitted to* α.

3.1.4 Category of actions

We define here the set *Act* of actions as follows.

$$\boldsymbol{\alpha = a(t_1, t_2, \ldots, t_n)|\phi?|\alpha_1 ; \alpha_2|\alpha_1 + \alpha_2}$$

where $a(t_1, t_2, .., t_n)$ is the atomic action. $\alpha_1;\alpha_2$ is the sequential composition of actions used to represent conjunction of requirements. $\alpha_1 + \alpha_2$ is the disjunction of actions and is used to represent disjunction of requirements. $\phi?$ is a test and it just verifies whether ϕ holds or not and is used to represent conditional requirements $(\phi?;\alpha) = (\phi\Rightarrow\alpha)$

3.2 Axioms

A0) T any tautology			
A1) $O(\phi \Rrightarrow \alpha)$	$\leftrightarrow$	$\phi \to O(\alpha)$	
A2) $P(\phi \Rrightarrow \alpha)$	$\leftrightarrow$	$\phi \to P(\alpha)$	
A3) $[\phi?]\psi$	$\leftrightarrow$	$\phi \to \psi$	
A4) $[\alpha_1;\alpha_2]\phi$	$\leftrightarrow$	$[\alpha_1]([\alpha_2])\phi$	
A5) $[\alpha_1+\alpha_2]\phi$	$\leftrightarrow$	$[\alpha_1]\phi \wedge [\alpha_2]\phi$	
A8) $[\alpha](\phi \to \psi)$	$\to$	$[\alpha]\phi \to [\alpha]\psi$	
A9) $\forall x\phi(x)$	$\to$	$\phi(t)$	Universal instantiation. x cannot be replaced by any variable which would become bound. That is, t is free in $\phi(x)$.
A10) $\forall x(\phi \to \psi)$	$\to$	$\phi \to \forall x\psi$	provided that x is not free in ϕ

3.3 Inference rules

a) Modus Ponens	**(MP)**	:	If ϕ and $\phi \to \varphi$ then φ
b) Necessitation rule for actions	**(Nec)**	:	If ϕ then $[\alpha]\phi$
c) Universal generalization	**(UG)**	:	If ϕ then $\forall x\, \phi$ provided x is not free in ϕ

The following are derived rules, which operate between actions, obligatory or not. *Pre* usually indicates pre-conditions and *Pos* post-conditions. Hoare's triple ($Pre\{\alpha\}Pos$) will be represented by the expression $Pre \to [\alpha]Pos$ in L_{DL} (See Harel, 1979; Gries, 1983, Hoare, 1969). The equality $Pre\{\alpha\}Pos = Pre \to [\alpha]Pos$ restricts both sides to hold in the same context.

(I;):	If $\vdash Pre \to [\alpha_1]Pos'$ and $\vdash Pos' \to [\alpha_2]Pos$ then $\vdash Pre \to [\alpha_1;\alpha_2]Pos$
(I+):	If $\vdash Pre \to [\alpha_1]Pos$ and $\vdash Pre \to [\alpha_2]Pos$ then $\vdash Pre \to [\alpha_1+\alpha_2]Pos$
(I$\Rrightarrow$):	If $\vdash (Pre \wedge \phi) \to [\alpha]Pos$ and $\vdash (Pre \wedge \neg\phi) \to Pos$ then $\vdash Pre \to [\phi?;\alpha]Pos$
(WP):	If $\vdash Pre \to [\alpha]Pos$ and $\vdash Pos \to \gamma$ then $\vdash Pre \to [\alpha]\gamma$
CDR 1:	If $\vdash Pre \to [\alpha]Pos$ and $\vdash P' \to [\alpha]Q'$ then $\vdash (Pre \wedge P') \to [\alpha](Pos \wedge Q')$
CDR 2:	If $\vdash Pre \to [\alpha]Pos$ and $\vdash P' \to [\alpha]Q'$ then $\vdash (Pre \vee P') \to [\alpha](Pos \vee Q')$

3.4 Semantics

It is set out here a possible world semantic for L_{DL} where W is the universe of all possible states or worlds. The semantic will be defined in a structure, which provides meaning for each constant, variable, function and predicate symbol. Formally, a model $\mathcal{M}$ is defined to be the structure $\langle W, D, Val, \Lambda, \nu, \tau, \kappa \rangle$, where

$W = \{w_0, w_1, \dots w_n, \dots\}$	W is a set of worlds or states
D	is a non empty set called domain.
Val	is a function assigning a semantic value to each non-logical constant of L_{DL}. That is, to standard constants, functions, predicates and actions.
$\Lambda: AtAct \times D^n \Rightarrow 2^{W \times W}$	Λ defines a set of pairs (w, w') describing actions $\alpha(d^n)$ such that starting in w the occurrence of the action would lead to the state w', where $d^n = (d_1, d_2, \dots d_n)$ and $d_i \in D$. (i=1, n).
ν	ν is a valuation that assigns a semantic value (*true* or *false*) to predicates and formulae.
τ	τ is the environment function that assigns to each variable x and world w an element d from D. τ is defined in the next Section below.
κ	κ provides the valuation for terms by using the environment function τ and *Val*.

3.4.1 Semantics for non-logical constants

According to the different kinds of constants in L_{DL}, *Val* assigns values to constant as follows.

- If c is a standard constant then $Val(c) = c_{Val} = d$ where $d \in D$.
- If p is a predicate constant then $Val(p) = p_{Val}$ where $p_{Val} \subseteq D^n$.
- If a^n is a n-ary action constant then $Val(a^n) = a^n_{Val}$ where $a^n_{Val} \subseteq W \times W \times D^n$.
- If f^n is a function from D^n to D, $Val(f^n) = f^n_{Val}$ where $f^n_{Val} \subseteq D^{n+1}$.

3.4.2 Semantics for terms

a) Environment function

Let τ be the semantic function for variables defined as follows. τ: $V \times W \Rightarrow D$, such that $\tau(x/d, w)$ is exactly like τ, except that $\tau(x/d, w)$ assign d to x in w.

b) Semantic for terms

- If x is a variable symbol then $\kappa(x)_{\tau,w}=\tau(x, w)=d$ such that $d \in D$.
- If c is a constant symbol then $\kappa(c)_{\tau,w}= Val(c)=d$ such that $d \in D$.
- If $f(t_1, t_2, .., t_n)$ is a function symbol from D^n to D, then

$$\kappa(f(t_1, t_2, .., t_n))_{\tau,w} = Val(f)(\kappa(t_1)_{\tau,w}, \kappa(t_2)_{\tau,w}, .., \kappa(t_n)_{\tau,w})$$

3.4.3 Semantics for actions

The semantics for actions is defined as follows.

- Atomic action:

If $a(t_1, t_2, .., t_n)$ is an atomic action then

$\Lambda(a(t_1, t_2, .., t_n))_{\tau,w} = \{(w, w')| (w, w') \in Val(a) (\kappa(t_1)_{\tau,w}, \kappa(t_2)_{\tau,w}, .., \kappa(t_n)_{\tau,w})\}$

- Tests: For any wff ϕ free of modalities:

$\Lambda(\phi?)_{\tau,w} = \{(w, w)| \nu(\phi)_{\tau,w} = true \}$

- Composition of actions:

$\Lambda(\alpha_1;\alpha_2)_{\tau,w}=\Lambda(\alpha_1)_{\tau,w}\circ\Lambda(\alpha_2)_{\tau,w}=\{(w, w')|\exists w''$ such that $(w, w'')\in \Lambda(\alpha_1)_{\tau,w}$ and $(w'', w')\in \Lambda(\alpha_2)_{\tau,w''}\}$

- Disjunction of actions:

$\Lambda(\alpha_1 + \alpha_2)_{\tau,w} = \Lambda(\alpha_1)_{\tau,w} \cup \Lambda(\alpha_2)_{\tau,w} = \{(w, w')| (w, w')\in \Lambda(\alpha_1)_{\tau,w}$ or $(w, w')\in \Lambda(\alpha_2)_{\tau,w}\}$

In L_{DL} the assumption that an obligatory actions is satisfiable can be expressed as $O(\alpha)\rightarrow<\alpha>true$. If *Pre* and *Pos* are the pre-conditions and post-conditions respectively of an action α, then partial correctness of α can be expressed by $Pre\rightarrow[\alpha]Pos$ and total correctness by $Pre\rightarrow<\alpha>Pos$ (See Harel 1979 and Gries, 1983).

3.4.4 Correctness of actions

An action α is partially correct with respect to a state w iff $w\models Pre\rightarrow[\alpha]Pos$. α is totally correct with respect to a state w iff $w\models Pre\rightarrow<\alpha>Pos$. Here the term correctness corresponds to the term used by Hoare (1969) for programs and that we use here for actions. Thus, $Pre\rightarrow[\alpha]Pos$ means that "If the assertion *Pre* is true before initiation of the action α, then the assertion *Pos* will be true on its completion" Hoare (1969: p. 577). When an action is totally correct it means that it is possible to perform that action, in other words, it is not an impossible action.

3.4.5 Encapsulation

In Dynamic Logic as presented by Harel (1979) and using Hoare's insights, if α is an action, α is correct iff $[\alpha]\psi$ holds in some state. Nevertheless given the correctness of an action $[\alpha_1]Pos_1$ it is possible to infer the correctness of a disjunction of actions $[\alpha_1+\alpha_2]Pos$. In this way if we do not make explicit which actions are obligatory and which are not, we would derive wrong conclusions. Thus, we encapsulate in a set all these actions which are obligatory or permitted. In this way we can have expression such as $O(\alpha_1) \wedge [\alpha_1+\alpha_2]Pos$ which says that α_1 is obligatory and that the disjunction of the obligatory action α_1 with α_2 ($[\alpha_1+\alpha_2]Pos$) is correct, but not that $\alpha_1+\alpha_2$ is obligatory.

3.4.6 Defining sets of obligatory actions

Here it is implemented the idea of encapsulating what is obligatory and permitted by defining subsets of *Act* as follows. This idea of distinguishing what is ordered can be observed also in Segerberg (1990) and Piwek (2000; 2001).

Definition: *Permitted and Obligatory requirements*

Let be *PAct* and *OAct* the sets of permitted and obligatory requirements (prescribed actions) respectively such that $OAct \subseteq PAct \subseteq Act$. These sets contains all actions which are permitted and obligatory respectively arranged in each possible world.

The sets of permitted and obligatory actions at a world can be defined as follows.

Definition: *Permitted actions at a world*

Let be $PAct_w = \{\alpha |\ \alpha \in PAct$ and α is permitted in $w\}$

Definition: *Obligatory actions at a world*

Let be $OAct_w = \{\alpha |\ \alpha \in OAct$ and α is obligatory in $w\}$

OAct can be expressed as the union of all obligatory actions at each world w in W as follows. $OAct = \cup_{w \in W} OAct_w$. Analogously *PAct* can be expressed as the union of all permitted actions at each world $w \in W$, $PAct = \cup_{w \in W} PAct_w$. The following conditions apply on the set $OAct_w$ and $PAct_w$ respectively.

C1) If $\alpha_1 \in OAct_w$ and $\alpha_2 \in OAct_w$ then $\alpha_1;\alpha_2 \in OAct_w$ or $\alpha_2;\alpha_1 \in OAct_w$

C2) If $\alpha_1 \in PAct_w$ and $\alpha_2 \in PAct_w$ then $\alpha_1;\alpha_2 \in PAct_w$ or $\alpha_2;\alpha_1 \in PAct_w$

C1 indicates that if α_1 and α_2 are obligatory then both are obligatory. $\alpha_1;\alpha_2$ and $\alpha_2;\alpha_1$ simply cover the case of dependent actions. C2) is the analogous of C1) for permission.

3.4.7 Semantics expressions in the language L_{DL}

a) Semantics for wff in L_{DL}

- $\mathcal{V}(p(t_1, t_2, .., t_n))_{\tau,w} = true$ iff $<\kappa(t_1)_{\tau,w}, \kappa(t_2)_{\tau,w}, .., \kappa(t_n)_{\tau,w}> \in Val(p)$.

  $\mathcal{V}$ defines the set of states where the predicate $p(t_1, t_2, .., t_n)$ is *true* and $Val(p) \subseteq D^n$.
- $\mathcal{V}(t_1 = t_2)_{\tau,w}$ = *true* iff $\kappa(t_1)_{\tau,w} = \kappa(t_2)_{\tau,w}$
- $\mathcal{V}(\neg\phi)_{\tau,w}$ = *true* iff $\mathcal{V}(\phi)_{\tau,w}$ = *false*
- $\mathcal{V}(\phi_1 \wedge \phi_2)_{\tau,w}$ = *true* iff $\mathcal{V}(\phi_1)_{\tau,w}$ = *true* and $\mathcal{V}(\phi_2)_{\tau,w}$ = *true*
- $\mathcal{V}(\exists x\phi)_{\tau,w}$ = *true* iff There exists an element d in D such that $\mathcal{V}(\phi)_{\tau(x/d,w), w}$ = *true*

  $\tau(x/d,w)$ is exactly like τ except that $\tau(x/d,w)$ assigns d to x.

b) Semantics for modal formulae

- $\mathcal{V}([\alpha]\phi)_{\tau,w}$ = *true* iff For every w' (if $(w, w') \in \Lambda(\alpha)_{\tau,w}$ then $\mathcal{V}(\phi)_{\tau,w'}$ = *true*)
- $\mathcal{V}(<\alpha>\phi)_{\tau,w}$ = *true* iff There exists $w'|\ ((w, w') \in \Lambda(\alpha)_{\tau,w}$ and $\mathcal{V}(\phi)_{\tau,w'}$ = *true*)

c) Obligatory and permitted actions

- $\mathcal{V}(O(\alpha))_{\tau,w}$ = *true* iff $\alpha \in OAct_w$
- $\mathcal{V}(P(\alpha))_{\tau,w}$ = *true* iff $\alpha \in PAct_w$

3.5 Notation (Truth with respect to a state)

Truth at state w of an arbitrary formula ϕ under L_{DL} for any valuation τ and the model $\mathcal{M}$ is inductively defined using the notation $w \in \mathcal{V}(\phi)_{\tau,w}$ and simply abbreviated as $w \vDash \phi$. When ϕ is not true at w under L_{DL} we can write $w \nvDash \phi$. If ϕ is valid in $\mathcal{M}$ we write simply $\vDash \phi$. We also use the notation $\xrightarrow{\alpha}$ as in Stirling (1992). According to the semantics just defined in the previous Section, this notation corresponds to the following expressions.

- $\mathcal{V}(\phi)_{\tau,w}$ = *true* is denoted by $w \vDash \phi$
- $(w, w') \in \Lambda(\alpha)_{\tau,w}$ is denoted by $w \xrightarrow{\alpha} w'$

Following Harel (1979), and adopting a free usage of conventional logical symbols and for a fixed state w, any formulae ϕ, ϕ_1 and ϕ_2 in *FOR*, α in *Act* and variable x, the following notation is used.

- $w \vDash p(t_1, t_2, .., t_n)$ iff $<\kappa(t_1)_{\tau,w}, \kappa(t_2)_{\tau,w}, .., \kappa(t_n)_{\tau,w}> \in Val(p)$
- $w \vDash \neg\phi$ iff it is not the case that $w \vDash \phi$
- $w \vDash (\phi_1 \wedge \phi_2)$ iff $w \vDash \phi_1$ and $w \vDash \phi_2$

- $w \models \exists x\phi$ iff there exists an element d in D such that $\nu(\phi)_{\tau(x/d), w} = true$
- $w \models [\alpha]\phi$ iff For every w'(if $w \xrightarrow{\alpha} w'$ then $w' \models \phi$)
- $w \models <\alpha>\phi$ iff There exists w'| ($w \xrightarrow{\alpha} w'$ and $w' \models \phi$)
- $w \models O(\alpha)$ iff $\alpha \in OAct_w$
- $w \models P(\alpha)$ iff $\alpha \in PAct_w$

3.6 Obligation with respect to context

Above we have defined obligation of an action with respect to a state or world. On the other hand if we have a description (set of statements) of a context, we can define the obligation of the action with respect to that context as follows.

Context is a collection of consistent propositions. Let $k=\{\phi_0, \phi_1, .., \phi_n\}$ represent our context, where $\phi_i \in FOR$. We may also identify the set of states defined by our context as follows. $\nu'(k) = \{w$| For every $\phi \in k$, $w \models \phi\}$

Now we can define obligation with respect to a context. If k represents a context and $\phi \in FOR$, we use the symbol '$\models$' and the notation $k \models_{\mathcal{M}} \phi$ to indicate that ϕ is *true* in the model $\mathcal{M}$ with respect to context k, for any assignment τ. We abbreviate $k \models_{\mathcal{M}} \phi$ simply as $k \models \phi$. When ϕ is *not true* at k under L_{DL} we can write $k \not\models \phi$. Thus,

- If $\phi \in FOR$, $k \models \phi$ iff For every w if $w \in \nu'(k)$ then $w \models \phi$

Using the notation above, $k \models O(\alpha)$ means that α is obligatory in context k.

3.7 Soundness

The only axiom different from dynamic logic that we need to verify to see that L_{DL} is sound is OPA).

Lemma: OPA) is sound under the restriction that $OAct_w \subseteq PAct_w$, for every state w in W.

Proof: Straightforward from the semantics of $O(\alpha)$ and $P(\alpha)$.

4. Obligation/Permission and inference

4.1 Jorgensen's dilemma

As we can see L_{DL} combines obligatory actions and propositions in inference. The analogous of Jorgensen's dilemma would be as follows.

If $p(x)$ = *x is your neighbour* and $\alpha(x)$ = *Love x as yourself*, then we can write the deontic version of the dilemma as in 1) which is assumed below.

1)	$k \models \forall x\, O(p(x) \Rightarrow \alpha(x))$ = *It is obligatory to love your neighbour as yourself*	
2)	$k \models \forall x\, p(x) \rightarrow O(\alpha(x))$	1), axiom A1)
3)	$k \models p(Alison)$ = *Alison is your neighbour*	assumption
4)	$k \models p(Alison) \rightarrow O(\alpha(Alison))$	2), Univ. Inst.
5)	$k \models O(\alpha(Alison))$= *It is obligatory to love Alison as yourself*	3), 4) and MP

This shows that L_{DL} mixes obligatory action with statements and allows making inference without being affected by inferential problems.

4.2 Paradox of Free Choice Permission-FCP

There is not rule to infer the membership of a disjunction of actions from the membership of one action in the set of obligatory or permitted actions. This is reinforced by the condition C1). That is, from $O(\alpha_1)$ is not possible to infer $O(\alpha_1+\alpha_2)$, for some other action α_2 and from $P(\alpha_1)$ is not possible to infer $P(\alpha_1+\alpha_2)$. Thus, L_{DL} does not validate the following inferences.

$O(\textit{Post the letter}) \rightarrow O(\textit{Post the letter} + \textit{Burn the letter})$
$P(\textit{Talk to the president}) \rightarrow P(\textit{Talk to the president} + \textit{Shoot the president})$

Therefore the model is not affected by the paradox of FCP.

4.3 Conjunction elimination and obligations

It is not possible to eliminate an action from a conjunction of either obligatory or permitted actions. That is, from $O(\alpha_1;\alpha_2)$ is not possible to infer either $O(\alpha_1)$ or $O(\alpha_2)$ and from $P(\alpha_1;\alpha_2)$ is not possible to infer either $P(\alpha_1)$ or $P(\alpha_2)$. This solves the problem of incomplete satisfaction derived in some approaches akin to the standard solution. For instance, if we represent the obligation of the imperative *Buy oranges and apples*!, as O(*Buy oranges* ; *Buy apples*), the model does not validate neither

$O(\textit{Buy oranges} ; \textit{Buy apples}) \rightarrow O(\textit{Buy oranges})$ nor
$O(\textit{Buy oranges} ; \textit{Buy apples}) \rightarrow O(\textit{Buy apples})$

We can observe that the model describes properties of imperatives such as the lack of truth-values and dependence. For instance the model may validate O(*Write a letter* ; *send the letter to your family*) but not O(*send the letter to your family* ; *Write the letter*). Examples using the operator for permission are analogous.

5. Conclusions

L_{DL} is given under first order dynamic logic, the operators for obligation and permission operate over actions rather that over statements. L_{DL} models the deontic concepts of obligation and permission not what it is uttered but what it is obligatory and permitted. Obligation can be verified with respect to context.

L_{DL} associates actions with imperatives, therefore it is not committed to assign truth-values to these sentences. Rather, it evaluates the membership of the action in turn in the set of obligatory actions. L_{DL} combines obligatory actions and propositions in inference. The model deals satisfactorily with the deontic version of Jorgensen's dilemma. It is not possible the elimination of an action from a conjunction of either obligatory or permitted actions. There is not rule to infer the membership of a disjunction of actions from the membership of one action in the set of obligatory or permitted actions. This avoids the paradox of FCP.

One of the main differences between the model presented here and those presented by Segerberg (1990) and Piwek (2000; 2001) is that we do not use descriptions to define actions but actions themselves and therefore classical logic does not permeate in our model avoiding so counterintuitive conclusions.

There are various applications for imperatives and deontic concepts, in particular, Meyden (1996), mentions that the deontic modalities are becoming increasingly of wide interest in computer science, with proposed applications including intelligent legal information systems, computer security, software engineering, database integrity constraints and agent oriented programming. Martino (1981) provides more examples of applications related to information retrieval, databases and legal information systems. In a more ambitious plan, he proposes that the public administration might be automated on the basis of verification and application of regulations. Recently agents have become quite popular. For instance Piwek (2000; 2001) models imperatives within a framework for communicating agents. Vere and Bickmore (1990) report the constructions of a basic agent. They affirm that a person can make statements to the agent, ask it questions, and give it commands. Thus, a robot receiving an order might be able to assess the order before making decisions towards the satisfaction of the order. This kind of autonomous behaviour demands an automatic interpretation of commands in which wrong obligations are not derived.

References

Alchourrón Carlos, Martino Antonio, 1990. "*Logic Without Truth.*" Ratio Juris. March 1990. 46-67.
Bunt, Harry. 2000. "*Dialogue pragmatics and context specification*" in "*Abduction, Belief and Context in Dialogue*;" Studies in Computational Pragmatics, Amsterdam: Benjamins, Natural Language. Processing Series No. 1, 2000. P. 81-150.
Buvac, Sasa. 1995. "*Resolving Lexical Ambiguity Using a Formal Theory of Context*." Visited in October 1998 in http://www-formal.Stanford.EDU/buvac/
Chellas, B., 1971. "*Imperatives.*" Theoria. Vol 37, 114-129. 1971

Gries, David, 1983. "*The Science of programming.*" Department of Computer Science. Cornell University. Upson Hall Ithaca, NY. 1983.
Hamblin, C. L, 1987. "*Imperatives.*" Basil Blackwell. USA. 1987
Hare R. M., 1961. "*The Language of Morals.*" Oxford at the Clarendon Press. Reprinted in 1961.
Harel David, 1979. "*First-Order Dynamic Logic.*" Lecture Notes in Computer Science. Edited by Goos and Hartmanis. 68. Springer-Verlag.Yorktown Heights, NY. 1979.
Hoare. C. A. R., 1969. "*An Axiomatic Basis for Computer Programming.*" Communications of the ACM, Vol. 12, No 10. October 1969. pp. 576 -580, 583.
Huntley Martin, 1984. "*The Semantics of English Imperatives.*" Linguistics and Philosophy. Vol 7. 1984. 103-133.
Jorgensen Jorgen, 1937. "*Imperatives and logic.*" Erkenntnis. Vol. 7, (1937-1938), pp. 288-296.
Kamp, H., 1973. "*Free-Choice Permission.*" Chicago: University of Chicago Press. Kamp, H. 1973. Proc. of the Aristotelian Society, N.S. 74: 57-74.
Levinson Stephen C., 1983. "*Pragmatics.*" Cambridge textbooks in linguistics. Cambridge University Press. 1983.
Lyons John, 1968. "*Introduction to Theoretical Linguistics.*" Cambridge at the University Press. 1968.
MacFadyen Heather, 1996. "*Using Verb Moods.*" HyperGrammar. University of Ottawa. 1996.
Manara and De Roeck, 1997. "*Context as Partial Beliefs, and the Pragmatic Modelling of Presuppositions.*" Context 97. Brazil. 66-74.
Martino A. Antonio, 1981. "*Deontic logic, Computational linguistics and legal information systems.*" Proceedings of the first international conference on logic, informatics, law. Vol. 2. Florence, Apr. 6-10. 1981. http://www.idg.fi.cnr.it/pubblicazioni/monografie/lid1bint.htm.
Megginson David, 1996. "*The Purpose of a Sentence.*" HyperGrammar. University of Ottawa. 1996.
Meyden R. van der, 1996. "*The Dynamic Logic of Permission.*" Journal of Logic and Computation, Vol 6, No. 3 pp. 465-479, 1996. A version of this paper appeared at the IEEE Symposium on Logic in Computer Science, Philadelphia, 1990.
Dignum F., Meyer J.-J.Ch., and Wieringa R.J., 1996. "*Free choice and contextually permitted actions.*" Studia Logica, 57:193-220, 1996.
Nodine Mark H., 1996. "*Glossary of Grammatical Terms.*" A Welsh Course. 1996. http://www.cs.brown.edu/fun/welsh/Glossary_main.html#I
Pérez-Ramírez, M., 2000. "*Imperatives, state of the art.*" CLUK 3. University of Brighton. UK. April, 2000.
Pérez-Ramírez, M., 2002. "*Formal pragmatic model for imperatives interpretation*" University of Essex. UK. (draft thesis).
Piwek, P., 2000. "*Imperatives, Commitment and Action: Towards a Constraint-based Model.*" In: LDV Forum: Journal for Computational Linguistics and Language Technology, Special Issue on Communicating Agents, 2000.
Piwek, P., 2001. "*Relating Imperatives to Action*." In: Bunt, H. and R.J. Beun, Cooperative Multimodal Communication, Lecture Notes in Artificial Intelligence Series 2155, Springer, Berlin/Heidelberg. 2001.
Radford Andrew, 1997. "*Syntactic theory and the structure of English. A minimalist approach.*" Cambridge University Press. 1997.
Ramsay, A. 2000. "*Speech act theory and epistemic planning*" in "*Abduction, Belief and Context in Dialogue*;" Studies in Computational Pragmatics, Amsterdam: Benjamins, Natural Language. Harry Bunt, Bill Black (eds). Processing Series No. 1, 2000. P. 293-310.
Ross A., 1941. "*Imperatives and Logic.*" Theoria (journal). Vol. 7. 53-71. 1941.
Segerberg Krister, 1990. "*Validity and Satisfaction in Imperative Logic.*" Notre Dame Journal of Formal Logic Volume 31, Number 2, Spring 1990. 203-221.
Sperber Dan and Wilson Deirdre, 1986. "*Relevance.*" Communication and Cognition. Great Britain London. 1986.
Stirling, Colin. 1992. "*Modal and temporal logics*." Handbook of Logic in Computer Science, vol. 2. Edit. S. Abramsky and D. Gabbay and T. Maibaum. Publisher, Oxford University Press. 477-563. 1992
Turner Raymond. 1992. "*Properties, Propositions and Semantic Theory. In Computacional Linguistics and Formal Semantics.*" Edited by Michael Rosner and Roderick Johnson.Cambridge University Press. Cambridge. 159-180. 1992
Vere, Steven and Bickmore, Timothy, 1990. "*A Basic Agent.*" Computational Intelligence, Vol. 6,
von Wright George Henrik, 1968. "*An Essay in Deontic Logic and The General Theory of Action.*" North Holland Publishing Company-Amsterdam. 1968.
von Wright, Henrik Georg. 1991. "*Is There a Logic of Norms?*" Ratio Juris. 1991. 67-79. 1991
von Wright, Henrik Georg. 1999. "*Deontic Logic: A personal View*" Ratio Juris. 1999. 26-38. 1999
Walter Robert, 1996. "*Jorgensen's Dilema and How to Face It.*" Ratio Juris. Vol 9. No. 2 June 1996. 168-71.

Weinberger Ota, 1991. “*The Logic of Norms Founded on Descriptive Language.*” Ratio Juris. 1991. 284-307.
Wieringa R.J. and Meyer J.-J.Ch.. 1993. “*Applications of deontic logic in computer science: A concise overview.*” In J.-J.Ch. Meyer and R.J. Wieringa, editors, Deontic Logic in Computer Science: Normative System Specification. Wiley, 1993. 17-40.

Acknowledgements

The first author would like to thank CONACYT-IIE.