

**OBLIGATIONS AND PERMISSIONS**  
**(AN INFORMAL INTRODUCTION TO STANDARD DEONTIC LOGIC)**

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ABSTRACT. Notes on an introduction to the idea of Deontic Logic and some of the basic issues and ideas.

Refer to SEP article for more details

<http://plato.stanford.edu/entries/logic-deontic/>

1. BACKGROUND

- (1) **What's it all about:** Permissions and Obligations in Linguistics (modalities, imperatives, interpretation of commands and legal documents, ...), Computer Science (specification theory, the meaning of programs, command and action systems, ...) and Philosophy (Moral philosophy, practical reasoning, ...). Will concentrate on linguistic examples.
- (2) **Paradigm.** The meaning of language. What do (e.g.) statements mean (e.g. true/false — is that adequate [no])? How do we represent and evaluate meaning? What are the legitimate inferences that can be drawn from and between statements? When are statements consistent with each other? What do statements *presuppose*, and how do we reason with presuppositions? How do we model anaphora, ellipsis, ambiguity, intentionality, tense, modality etc. at the level of (representations of) “meaning”?

- (3) **Basic obligations/permissions.**

- (a) “*You should pay your rent on time.*”
- (b) “*You should help strangers in trouble.*”
- (c) “*You must not walk on the grass.*”
- (d) “*Pay your taxes.*”
- (e) “*You may walk on the grass.*”
- (f) “*You may pick flowers.*”

What do they mean? Are they like propositions? What is their “content”? How do we evaluate them? What does it mean to comply with an obligation? How are obligations related to permissions?

- (4) **Complex obligations/permissions.**

- (a) “*You may sleep in the bed or on the sofa.*”
- (b) “*You should pick the carrots and the potatoes.*”
- (c) “*You should take the train now, if you want to be on time.*”

How are the meanings of the parts (of the arguments) related to the whole? What kinds of inferences are supported between obligations/permissions with arguments that “entail” each other?

- (a) “*You should help someone who is hurt.*”
- (b) “*John is hurt.*”
- (c) “*You should help John.*”

- (5) **Some key issues** (a summary). How do we evaluate obligations and permissions? What is the nature of the “argument” to an obligation/permissive statement (e.g. is it (also) a proposition)?

## 2. STANDARD DEONTIC LOGIC (SDL)

- (6)
- Introduction to notation.**
- Letters for sentences and for arguments to obligations.

 $p, q, r, \dots$ 

Logical operators

 $\wedge, \vee, \rightarrow, \neg$ Modal operators for obligation (e.g.  $O$ ) and permission (e.g.  $P$ ), with sentences as arguments. $Op, Pq$ Representation of obligation ( $Oa$ ), permissible ( $Pa$ ), impermissible ( $\neg Pa$ ) forbidden ( $O\neg p$ ).

- (7)
- Standard Deontic Logic.*
- Rules and axioms.

**TAUT:** All tautologous wffs of the language**OB-K:**  $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$ **OB-D:**  $Op \rightarrow \neg O\neg p$ **MP:** If  $\vdash p$  and  $\vdash (p \rightarrow q)$  then  $\vdash q$ **OB-NEC:** If  $\vdash p$  then  $\vdash Op$ Effectively, this gives rise to a system where the *arguments* of deontic operators are subject to the same entailment patterns as the propositional language (see OB-RM below). Should this be the case?

- (8) **A Model.** Why do we need a model? To give an account of the (intended) interpretation of deontic statements. To help demonstrate that there is a consistent interpretation of the theory.
- (9) **A possible worlds model.** Sketch of a “Utopian” possible worlds model: some worlds are “better” than others. The rules of logic apply in each world. Content of obligations are true propositions in (accessible) ideal worlds. That which is permitted is not false in (accessible) ideal worlds. There are numerous variations of this account. A presumed “meta-obligation”: to move to an ideal/better world? What is the relationship between action (to do) and proposition (to be)?
- (10) Relation to alternative alethic accounts (Andersonian/Kangorian) with a proposition characterising ideal/non-ideal worlds.

## 3. PROBLEMS WITH SDL

Some issues with logical consequence in SDL.

- (11) **Necessitation.**  
*“You are obliged to ensure that  $e^{i\pi} = -1$ .”*  
 In what sense is this an obligation?
- (12) **Free choice.**  
 (a) *“You are permitted to sleep on the sofa or the bed.”*  
 (b) *“You are permitted to sleep on the sofa and the bed.”*
- (13) **Derivability of OB-RM** If  $\vdash (p \rightarrow q)$  then  $\vdash (Op \rightarrow Oq)$ . This gives rise to a number of questions.
- (14) **Disjunction Introduction** (Ross’ Paradox)  
 (a) *“You should post the letter.”*  
 (b) *“You should post the letter or burn it [the letter].”* (Introduction of free choice — should that be allowed?)
- (15) **Conjunction Elimination** (distribution of obligation, and questions of partial fulfillment).  
 (a) *“You should jump and land on the train.”*  
 (b) *“You should jump.”* (What about partial fulfillment?)  
 (c) *“You should get the advice of a surgeon and amputate the arm.”*  
 (d) *“You should amputate the arm.”*
- (16) **Conjunction Introduction** (Schotch and Jennings).  
 (a) *“You should<sub>a</sub> go.” “You shouldn’t<sub>b</sub> go.”*  
 (b) *“You should go and not go.”*

Other issues with complex and conditional obligations:

- (17) **The Good Samaritan** (Prior).  
 (a) "It ought to be the case that Jones helps Smith who has been robbed."  
 (b) "It ought to be the case that Smith has been robbed."  
 (c) What about "You ought to use a clean knife."? (Fox)
- (18) **Contrary to duty obligations** (Chisholm's Paradox)  
 (a) "It ought to be that Jones does go (to the assistance of his neighbors)."  
 (b) "It ought to be that if Jones does go then he tells them he is coming."  
 (c) "If Jones doesn't go, then he ought not tell them he is coming."  
 (d) "Jones doesn't go."

In the latter case, how do we formulate the conditionals. How to reason if an obligation is breached.

- (a)  $Og$   
 (b)  $O(g \rightarrow t)$   
 (c)  $\neg g \rightarrow O \neg t$   
 (d)  $\neg g$

Conclude  $Ot$  and  $O \neg t$ .

#### 4. SOME ALTERNATIVES

- (19) **Dyadic obligation**  $O(b/a)$ . Obligation is for  $b$  in the event of  $a$ . But how is this interpreted ( $a \rightarrow Ob$  "factual detachment" v.  $Oa \rightarrow Ob$  "deontic detachment")?
- (20) **Conditionals** How is  $\rightarrow$  properly interpreted in modal contexts? (e.g. Kratzer's modal subordination, Lewis' counterfactuals?)
- (21) **Alternative interpretations**. What happens to these problem cases if we consider the logic of the *satisfaction criteria* of obligations, rather than the obligations themselves? What is the proper nature of arguments to obligations and permissions (propositions v. "action specifications")?

#### 5. NEXT STEPS

- (22) Is it appropriate to consider *one* logic for deontic systems? How much is the justification of a particular stance on the axioms of SDL (etc.) dependent on particular interpretations of particular examples? Are there factors in the intuitions about examples in the deontic literature that are really to do with "other" phenomena (e.g. generics, presuppositions)?
- (23) **Hint at my contribution**. Reconsider some of the examples (e.g. The Good Samaritan and the "clean knife" example), and some of the example inferences and assumptions. Re-think what deontic logic is *about*, and which existing systems are closer in spirit to such considerations. Systems of authority. Allow contradictory authorities. Consider notions of satisfaction and presupposition (relevant for "Ought implies Can"?). Current work: a logic of transgressions.