Observer-based Delay Compensation for Networked Control Systems – analysis and synthesis

Octavian Stefan, Toma-Leonida Dragomir
Department of Automation and Applied Informatics
Politehnica University Timisoara
Timisoara, Romania
{octavian.stefan, toma.dragomir}@upt.ro

Alexandru Codrean
Department of Automation
Technical University of Cluj-Napoca
Cluj-Napoca, Romania
alexandru.codrean@aut.utcluj.ro

Abstract — The current study focuses on the stability of a generic observer-based delay compensation structure for networked control systems with time-varying delays. After a brief presentation of the design principles, the stability conditions are derived in terms of maximum delay bounds, using a Lyapunov functional. Experimental results validate the entire approach on a specific case study.

Keywords — networked control systems; time delay; disturbance observers; stability analysis;

I. INTRODUCTION

Networked control systems (NCS) gained an increasing attention from the scientific community in the last years because of the rapid development of communication technology ([1], [2]). Low cost, high performance and reliability make the NCS a viable solution for distributed control systems.

Although NCS have a lot of advantages over the conventional control systems, there are also some shortcomings induced by the network component like time varying delays, information loss, limited communication capacity; that tend to complicate the design and analysis phases of the NCS ([3]). Multiple control strategies have been developed by the scientific community in order to overcome these shortcomings. The most important of them are based on: robust control ([4]), optimal stochastic control ([5]), event based control ([6]), model predictive control ([7]), gain scheduling ([8]) and adaptive Smith predictor ([9]).

One solution of interest for time-varying delay compensation in NCS considers the transmission delay as an unknown additive disturbance at the input of the process [10]. The disturbance is estimated by using a communication disturbance observer (CDOB) and then its effect on the control system is filtered from the controller’s point of view. The main advantage of this approach is that it needs no a priori information about the time delay instantaneous values or variation speed.

Although several particular CDOB-based network control structures have been designed and analyzed in previous studies, the current study addresses the stability analysis and control synthesis of a generic observer-based delay compensation structure for the general case involving time-varying delays. In [10], [11] and subsequent studies the stability of a CDOB-based NCS was proven only for constant time delay values.

The remainder of this paper is organized as follows. Section II introduces the observer-based network control structure. Section III presents the NCS’s control design. Section IV analyses the NCS’s stability. Section V presents an illustrative example. Section VI states some final conclusions.

II. OBSERVER-BASED NETWORKED CONTROL STRUCTURE

The considered observer-based NCS structure, proposed in [12], is presented in Fig. 1. The aim of the networked control structure is to reject the disturbance effect of the network and of the local disturbance, in order to ensure the imposed process control behavior. At the local side containing the process, the disturbance d is compensated by a feedback loop composed of a disturbance observer (DOB) and a disturbance compensator (DCO). The controller, placed at the remote side, is separated from the process by the network, considered as a discrete-time nonlinear system.

As a design hypothesis, both channels of the network are modeled as time-varying delay elements and the process as linear time invariant (LTI). As consequence, first, the round trip time (RTT) delay of the network can be obtained by combining both time delay elements. Second, the effect of the RTT is considered as a delay disturbance dn, acting at input of the process, and defined as the difference between the transmitted control signal u and the received one u0 ([10])

\[ d_n(t) = u_0(t) - u(t) , \]

with

\[ u_0(t) = u(t - \tau) . \]

Fig. 1. Observer-based networked control structure
The current study presents the design principles for the NCS from Fig. 1, analysis the stability of the network control structure, and finally pursues an extensive validation, with respect to the network delay bounds which ensure stability.

### III. CONTROL DESIGN

Consider a LTI process of the form

\[
\begin{align*}
\dot{x}_p(t) &= A_p x_p(t) + b_p u_p(t) + b_{pd} d(t), \\
y_i(t) &= c_i^T x_i(t)
\end{align*}
\]

with \( x_p \in \mathbb{R}^p \), \( u_p \in \mathbb{R}, d \in \mathbb{R}, y_p \in \mathbb{R} \) and the output feedback controller

\[
\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) + b_c w(t) - y_i(t), \\
u(t) &= c_c^T x_c(t) + d_c y_i(t)
\end{align*}
\]

with \( x_c \in \mathbb{R}^c \) and \( w \in \mathbb{R} \).

The controller is designed as if it were directly connected to the process \( (u = u_p, y_i = y_p) \).

Next, consider the local feedback loop (DOB+DCO), with the disturbance \( d \) acting on the process. For slow variations in time of \( d \), a first order exogenous system, \( d(t) = 0 \), can be used in the design of the DOB. The observer’s parameters are then adopted such that the estimated disturbance converges to the real disturbance \( d \). The DCO is designed to ensure complete local disturbance compensation in steady state regime. The local feedback loop can be framed in the state space form as

\[
\begin{align*}
\dot{x}_a(t) &= A_a x_a(t) + b_a u_a(t) + b_{ap} y_p(t) \\
u_a(t) &= c_a^T x_a(t) + d_a y_p(t)
\end{align*}
\]

The CDOB provides an estimate for the network disturbance \( d_n \) defined as in (1). Because \( d_n \) is induced by digital network transmissions, it can be regarded as a staircase signal that can be obtained from a first order exogenous system \( d_n(t) = 0 \), which is then included in the CDOB. The CDOB parameters are chosen such that:

i) the transfer function that relates the output \( d_n \) to the input \( d \) should behave as a low pass filter with a sufficiently high cut-off frequency (i.e. \( d_n \rightarrow d_n \) for an imposed frequency domain);

ii) the transfer function that relates the output \( d_n \) to the input \( d \) should attenuate low frequency signal components in order to reject the residual error of the local compensation loop.

The CDOB can be described by the following state-space model

\[
\begin{align*}
\dot{x}_a(t) &= A_a x_a(t) + b_a u(t) + b_{ap} y_p(t) \\
d_n(t) &= c_a^T x_a(t) + d_a y_p(t)
\end{align*}
\]

As a final step, the CDOB is coupled with the process model

\[
\begin{align*}
\dot{x}_{mp}(t) &= A_p x_{mp}(t) + b_p d_n(t) \\
y_c(t) &= c_p^T x_{mp}(t)
\end{align*}
\]

in order to reject the delay disturbance from the controller’s point of view.

### IV. STABILITY ANALYSIS

In order to analyze the stability of the networked control structure from Fig. 1, first, three simplifying assumptions will be made:

\( A_1 \): The local disturbance \( d \) and the reference \( w \) are assumed to be null.

\( A_2 \): The network is idealized as a time-varying delay transfer element placed on the direct path, with the delay equal to the RTT. Consequently, the influence of data loss and the communication channels’ limited capacity are neglected (the packet loss rate is assumed to be negligible with respect to the RTT). Consequently, the influence of data loss and the communication channels’ limited capacity are neglected (the packet loss rate is assumed to be negligible with respect to the RTT).

\( A_3 \): The RTT delay is assumed to be bounded:

\[ 0 \leq \tau(t) \leq \tau_{\max} \]

Because \( w(t) = 0 \), the input to the controller \( C \) is \( -y_i(t) = -(y_s(t) - y_d(t)) \). As specified in assumption \( A_2 \), the network is replaced by a single time varying delay on the direct path (equal to the RTT), such that \( u_n(t) = u(t - \tau(t)) \) and \( y_n(t) = y_p(t) \).

The closed loop model of the NCS is given by

\[
\dot{x}(t) = Ax(t) + A_c x(t - \tau(t)),
\]

where \( x(t) = [x_i^T(t) \ x_c^T(t) \ x_{mp}^T(t) \ x_{mp}^T(t)]^T \) and

\[
A = \begin{bmatrix}
A_p & b_p d_n A_c & b_{ap} c_{p0} & 0 & 0 \\
b_{ap} c_{p0}^T A_p & b_{ap} d_n & b_{ap} d_n & b_{ap} c_{p0} & 0 \\
0 & b_{ap} d_n & A_p & -b_c c_{p0} \ A_p & 0 \\
-b_c c_{p0} & 0 & 0 & b_c c_{p0} & A_p \\
b_{ap} d_n & b_{ap} d_n & b_{ap} c_{p0} & 0 & b_c c_{p0} \\
0 & b_{ap} d_n & b_{ap} c_{p0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
The system (8) is now in standard form for time delay systems, for which several stability methods were investigated in the literature in the last two decades. The methods can be classified according to three categories: delay independent, delay dependent and rate dependent, delay dependent and rate independent. Because in practice the delay is usually bounded in a certain range \( (A_3) \), and the rate of the delay variation is usually unknown, here the focus will be on the delay dependent and rate independent case.

The aim is to find the maximum range \([0, \tau_{\text{max}}]\) in which the time delay can vary, for which the NCS is still stable. In other words, to find the maximum value of \( \tau_{\text{max}} \) for which the system is stable. To this end, the following theorem gives the sufficient conditions for stability for a given delay range \([0, \tau_{\text{max}}]\). The theorem, along with the proof, is an adaption of the results from [13].

**Theorem 1:** The system (8) with time-varying delay \( \tau(t) \) of upper bound \( \tau_{\text{max}} \) is asymptotically stable if there exist symmetric positive definite matrices \( P, Q, Z \), and matrices \( N_1, N_2, S_1, \) and \( S_2 \) such that the following linear matrix inequality (LMI) holds

\[
\begin{bmatrix}
PA + A'P + Q + N_1'PA + N_2' - N_1 - S_1 - S_2 - \tau_{\text{max}}N_1 - \tau_{\text{max}}S_1 - \tau_{\text{max}}Z
\end{bmatrix} = 0,
\]

where * stands for symmetric term in a symmetric matrix.

**Proof:** Consider the Lyapunov functional candidate ([13])

\[
V(x(\theta)) = x(I)(P)(x(t) + \int_{\tau_{\text{max}}}^{t} x(s)Q x(s) \, ds + \int_{\tau_{\text{max}}}^{t} x(s)Z x(s) \, ds d\theta,
\]

where \( P = P^T > 0, Q = Q^T > 0, Z = Z^T > 0 \) and \( x(\theta) = x(t + \theta) \), with \( -\tau_{\text{max}} \leq \theta \leq 0, t \in [t_{\text{a}}, t_{\text{b}}] \). \( C([t_{\text{a}}, t_{\text{b}}], \mathbb{R}^n) \) is a Banach space of continuous functions mapping the interval \([t_{\text{a}}, t_{\text{b}}]\) into \( \mathbb{R}^n \), with the norm \( \|x(\theta)\| \).

Based on the Leibniz-Newton formula, the following equations hold for any matrices \( N_1, N_2, S_1, \) and \( S_2 \) (weighting matrices)

\[
\begin{bmatrix}
2[x'(t)N_1 + x'(t - \tau(t))N_1]
\end{bmatrix} = 0,
\]

Additionally, the following equation also holds

\[
\begin{bmatrix}
\dot{x}(t) - x(t - \tau(t)) = 0
\end{bmatrix} = 0.
\]

By making use of Leibniz’s integral rule, the derivative of \( V(x(t)) \) along the solutions of (8) can be written as

\[
\dot{V}(x(t)) = 2x'(t)P\dot{x}(t) + x'(t)Qx(t) - x'(t - \tau_{\text{max}}).
\]

The further use of equations (12)-(14), and after some calculations and regrouping yield

\[
\begin{bmatrix}
\dot{V}(x(t)) \leq 2x'(t)P\dot{x}(t) + x'(t)Qx(t) - x'(t - \tau_{\text{max}})
\end{bmatrix} = 0,
\]

where

\[
\zeta(t) = [x(t - \tau(t)), N_1, S_1, \lambda = A_1', P + A'P + Q + N_1', PA_j + N_2' - N_1 - S_1 - S_2, *]
\]

The last two integral terms can be dropped, and (16) becomes

\[
\dot{V}(x(t)) \leq \zeta(t)[\Gamma + \lambda Z + \lambda Z^T] \zeta(t).
\]

The condition \( \Omega < 0 \) is equivalent to (10) by Schur complements. Thus, if (10) holds, then \( \dot{V}(x(t)) < -\varepsilon \|x(t)\|^2 \) for a
sufficiently small \( \varepsilon > 0 \), and as a result the system (8) is asymptotically stable.

V. CASE STUDY

Consider the networked control structure from Fig. 1, composed of an electric drive as the controlled process, a PI controller, a TCP/IP network, two Luenberger observers ([14]) and a non-inertial local disturbance compensator. As control objective, the motor speed \( \dot{y}_p \) must follow the reference \( w \), despite of the disturbances \((\text{load disturbance } d \text{ and network disturbance } d_n)\) affecting the control system.

A. Numerical setting

The electric drive is composed of a permanent magnet DC motor supplied by an electronic actuator (PWM) and a tachogenerator. The model associated with the electric drive has the form (3) with \( x_{\text{p}1} = [x_{\text{p}1} \ x_{\text{p2}}] \) (\( x_{\text{p}1} \) is the motor speed, \( x_{\text{p2}} \) is the armature current) and

\[
A_p = \begin{bmatrix} 0 & T_m^{-1} \\ -T_p^{-1} & -T_p^{-1} \end{bmatrix}, \quad b_p = \begin{bmatrix} 0 \\ T_m \end{bmatrix}, \quad c_p^T = [1 \ 0]. \tag{19}
\]

The model (4) of the PI controller is defined by

\[
A_i = 0, \quad b_i = 1, \quad c_i^T = K_a d_i = K_a. \tag{20}
\]

The local feedback loop (DCO+DOB) modeled by (5) has

\[
A_{\text{loc}} = \begin{bmatrix} -T_m + T_1 s^{-1} \quad 1_{a1} \\ -T_1 s^{-1} \quad 1_{a2} \end{bmatrix}, \quad b_{\text{loc}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad c_{\text{loc}} = \begin{bmatrix} 1 \ 1 \end{bmatrix}.
\]

\[
b_{\text{Loc}} = \begin{bmatrix} T_m \ T_1 s^{-1} \\ 1_{a1} s^{-1} - T_2 s^{-1} T_m \end{bmatrix}, \quad c_{\text{loc}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]

The DOB is a reduced order observer ([15]) and the DCO is the proportional element \( K_a \). Because the observer implies the change of variables \( x_{\text{d}} = x_{\text{loc}} + y_f \), with \( x_{\text{loc}} = [x_{\text{loc}1} \ x_{\text{loc}2}] \) and \( x_f = [x_{\text{loc1}} \ x_{\text{loc2}}] \), the output is actually \( u_1 = K_a \hat{d} \).

The CDOB is a full order observer of type (6) with

\[
A_o = \begin{bmatrix} -l_{a1} & T_m^{-1} & 0 \\ -T_a^{-1} - l_{a2} & -T_a^{-1} & T_a^{-1} \\ -l_{a3} & 0 & 0 \end{bmatrix}, \quad b_o = \begin{bmatrix} 0 \\ T_a^{-1} \\ 0 \end{bmatrix},
\]

\[
b_o = \begin{bmatrix} l_{a1} \\ l_{a2} \\ l_{a3} \end{bmatrix}, \quad c_o^T = [1 \ 0 \ 0] d_o = 0
\]

and the state vector \( x_1 = [\hat{x}_{\text{p}1} \ \hat{x}_{\text{p}2} \ \hat{d}] \).

All the parameters of the NCS are given in Table I.

The manner in which the design parameters were obtained will be disclosed in the next section.

B. Design principles

This section briefly presents the main design principles regarding the NCS. The design follows the steps explained in detail in [12].

A PI controller was chosen, which ensures a null steady state error, in order to show the robustness (with respect to the network disturbance) of the NCS from Fig. 1 with a simple (classic) controller. The controller parameters were obtained by imposing a time constant \( T_1 = K_p / K_i \) in order to compensate the inertia of the process and a gain \( K_p \) according to a desired settling time.

The design parameters of the local feedback loop composed of DOB and DCO are adopted in order to satisfy the static and dynamic requirements of the local closed loop system. Based on Fig. 1 and the models of the process, DOB and DCO, the estimated disturbance can be obtained as

\[
\hat{d}(s) = \frac{T_m l_{a2}(T_s s + 1)}{T_m T_s s^2 + (T_m + 1 s l_{a1} T_s - T_s T_m l_{a2}) s - l_{a2} T_m}. \tag{23}
\]

The process output \( y_p \) expressed as

\[
y_p(s) = H_1(s) u_p(s) + H_2(s) d(s)
\]

\[
= \frac{1}{T_1 T_s s^2 + T_s s + 1} u_p(s) - \frac{(1 + T_s) T_m}{T_1 T_s s^2 + T_s s + 1} u_1(s), \tag{24}
\]

with the aid of \( u_1(s) = u_1(s) + K_a \hat{d}(s) \), can be brought to

\[
y_p(s) = H_2(s) \left[ H_1(s) / H_1(s) \cdot u_1(s) + \left( H_1(s) K_a H_1(s) / H_2(s) + 1 \right) d(s) \right]. \tag{25}
\]

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>CONTROL SYSTEM'S PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>Pwr</td>
<td>20W</td>
</tr>
<tr>
<td>J*</td>
<td>5.18\times10^{-4} \text{ kg m}^2</td>
</tr>
<tr>
<td>( Y_{p \text{max}} )</td>
<td>4500 rpm</td>
</tr>
<tr>
<td>( T_m )</td>
<td>0.157 sec</td>
</tr>
<tr>
<td>( T_a )</td>
<td>0.039 sec</td>
</tr>
<tr>
<td>( K_p )</td>
<td>0.460</td>
</tr>
<tr>
<td>( K_i )</td>
<td>1.590</td>
</tr>
<tr>
<td>( K_a )</td>
<td>0.157</td>
</tr>
<tr>
<td>( l_{a1} )</td>
<td>-0.300</td>
</tr>
<tr>
<td>( l_{a2} )</td>
<td>-13.260</td>
</tr>
<tr>
<td>( l_{a3} )</td>
<td>102.040</td>
</tr>
<tr>
<td>( l_{a4} )</td>
<td>410.010</td>
</tr>
<tr>
<td>( l_{a5} )</td>
<td>461.430</td>
</tr>
</tbody>
</table>

* Pwr – motor’s power, ** J – moment of inertia
The static condition for disturbance rejection is that \( H_0(0)=0 \), which leads to \( K_a=T_m \). The parameters \( l_1 \) and \( l_2 \) are further adopted such that \( H_\alpha \) acts as a high pass filter (Fig. 2).

Next, a full order CDOB was preferred, instead of a reduced order one, because it leads to better estimation performance in practice, due to its filtering capacity ([12]). Starting from Fig. 1, the models for the process, DOB, DCO and CDOB, the estimated network disturbance can be calculated as

\[
\hat{d}_n(s) = H_n(s)d_n(s) + (T_1 T_2 s + T_3) H_n(s) H_n(s) d_n(s) \cdot \tag{26}
\]

with

\[
H_n(s) = \frac{-1}{T_1 T_2 s^2 + T_3 (1 + l_1 + l_2) s + (l_1 T_2 + l_2 + 1) s + 1} \cdot \tag{27}
\]

The conditions for which \( \hat{d}_n \rightarrow d_n \) are

\[
\begin{align*}
\text{Condition 1: } & H_n(j\omega) \rightarrow 1 \\
\text{Condition 2: } & H_n(j\omega) H_\beta(j\omega) \rightarrow 0 \\
\text{Condition 3: } & \text{CDOB stable \& faster than the process}
\end{align*}
\]

The first condition implies that \( H_n \) should have a magnitude close to unity for a frequency domain as large as possible (i.e. \( \hat{d}_n \) should be as close as possible to \( d_n \)). The second condition implies that across the imaginary axes \( H_n H_\beta \) should have a magnitude as small as possible (\( d_n \) should not influence the estimation of \( d_n \)). The third condition implies that the poles of the observer have negative real part, with absolute values larger than the process poles.

The design parameters \( l_1, l_2, l_3 \) were adopted by imposing a cut-off frequency \( \omega_n=37 \text{ rad/s} \) for \( H_n \) (Fig. 3). Note that a compromise has been made in order to avoid the peaking phenomenon due to large observer gains, respectively large cut-off frequencies (see [12] for discussion).

C. Stability assessment

Assessing the stability of the NCS, the LMI condition (10) from Theorem 1 is solved using CVX Toolbox for Matlab ([16]), by formulating the problem as a convex optimization one. Solutions of the optimization problem were found (matrices \( P, Q, Z, N_1, N_2, S_1 \) and \( S_2 \)) up to a maximum delay upper bound \( \tau_{\text{max}} = 0.35 \text{ s} \), thus proving according to Theorem 1 that the system is asymptotically stable. Although the stability conditions are less conservative than most presented in the literature ([13]), the result may still be conservative. However it is usually good enough in practice - a network delay range [0 s, 0.35 s] is consistent with most real life network transmission scenarios over the Internet ([17]).

D. Network characteristics

The network considered as reference for the NCS is a TCP/IP based wide area network (WAN). Under regular conditions a WAN is characterized by time varying delays ranging from a few tens to a few hundreds of milliseconds, the dominant component of the delay being the propagation delay ([17]).

When using an unreliable protocol for data transport (typically the case for a NCS transmission), the communication is also characterized by information loss, mostly because of packet drops due to network equipment saturation. Under such a scenario, there are mainly two approaches for maintaining the quality of control:

i) adopt a sufficiently small sample period in respect with the maximum packet loss rate and the process dynamics ([1]);

ii) impose a certain quality of service to the network by using an implementation aware co-design method (e.g. [18]).

The current case study considers the first approach, and as a result the network is characterized only by a time-varying delay defined by the network RTT.

E. Experimental results

The networked control structure from Fig. 1 was developed in a Matlab/Simulink environment and implemented on a dSPACE board connected to the electronic actuator and to the tachogenerator. The values of the time-varying delay (Fig. 4) were generated as uniformly distributed pseudo-random numbers. The delay varies between 0.05 s and 0.58 s with an average of 0.35 s (the scenario was chosen in order to show that the NCS can cope with extreme network transmission delay variations and values, which exceed the delay bound \( \tau_{\text{max}} \) provided by the sufficient stability conditions).

The experimental results show that the networked induced delays destabilize the system without the CDOB estimation (Fig. 5), leading to oscillations in the system’s response (the control objective is no longer met). The extension of the NCS with the CDOB structure manages to eliminate the oscillations induced by the network and assures good tracking performances (Fig. 6), proving the capability of the CDOB to estimate and reject the delay disturbance (Fig. 7).
VI. CONCLUSION

The current study addresses the design, analysis, and validation of the generic observer-based delay compensation structure from Fig. 1. First, the design principles are briefly presented, while the case study further particularizes the design. Second, the stability of the observer-based delay compensation structure is addressed for the general case involving time-varying delays. Sufficient stability conditions are given, in terms of maximum delay bounds, using a Lyapunov functional. The stability conditions remain the same, regardless of the design approach used for different types of NCS that can be framed into this structure. Finally, the case study validates through experiments the observer-based NCS for the entire delay range obtained from the stability conditions.

ACKNOWLEDGMENT

This work was partially supported by the strategic grant POSDRU/159/1.5/S/137070 (2014) of the Ministry of National Education, Romania, co-financed by the European Social Fund – Investing in People, within the Sectoral Operational Programme Human Resources Development 2007-2013.

REFERENCES