Obligations and Permissions

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Abstract

Utterances and statements that are concerned with obligations and permissions are known as deontic expressions. They can present something of a challenge when it comes to formalising their meaning and behaviour. The content of these expressions can appear to support entailment relations similar to those of classical propositions, but such behaviour can sometimes lead to counter-intuitive outcomes. Historically, much of the descriptive work in this area has been philosophical in outlook, concentrating on questions of morality and jurisprudence. Some additional contributions have come from computer science, in part due to the need to specify normative behaviour. There are a number of formal proposals that seek to account for obligations and permissions, such as Standard Deontic Logic. In the literature, there has also been discussion of various conundrums and dilemmas that need to be resolved, such as the Good Samaritan, the Knower, the Gentle Murderer, Contrary to Duty Obligations, Ross’s Paradox, Jørgensen’s Dilemma, Sartre’s Dilemma, and Plato’s Dilemma. Despite all this work, there still appears to be no definite consensus about how these kinds of expressions should be analysed, or how all the deontic dilemmas should be resolved. It is possible that obligations themselves, as opposed to their satisfaction criteria, do not directly support a conventional logical analysis. It is also possible that a linguistically informed analysis of obligations and permissions may help to resolve some of the deontic dilemmas, and clarify intuitions about how best to formulate a logic of deontic expressions.
1 Introduction

Work on the formal semantics of natural language has often focused on the propositional interpretation of indicative sentences. Such sentences can be analysed in terms of their truth conditions. This is achieved by translating sentences into propositions expressed in some form of classical logic. The logic may be enriched to make it easier to deal with phenomena such as anaphora and propositional attitudes, like belief and knowledge, and the modalities of necessity and possibility.

This article is concerned with aspects of the interpretation of obligations and permissions. Statements pertaining to obligations and permissions are called deontic expressions.\(^1\) Logical systems that set out to capture the inferential behaviour of such expressions are referred to as deontic logic.

1.1 Deontic Expressions

Basic examples of deontic expressions include those given in (1). Some more complex cases will be considered later (for example, in §4).

(1) (a) “Peter must close the door.”

(b) “Mary is obliged to find a job.”

(c) “You must pay your taxes.”

(d) “You can walk on the grass.”

(e) “You are permitted to delay payment for up to three months.”

(f) “Evan may go to the beach.”

Deontic expressions do not necessarily include words directly related to “obligation” or “permission”, but instead can employ a modal verb, such as “must”, “should”, “can”, “may”, among others. These modal verbs modify the intended interpretation of the underlying propositional content so that it is no longer a simple factual claim. There can be some ambiguity in the precise nature of the meaning of such modal expressions. For example, “can” may be used to express a physical ability\(^2\) rather than permission.

(2) “John can run very fast.”
And “should” and “ought” may be used to express epistemic claims (3), claims that make predictions based on our knowledge and belief.

(3) (a) “The coin should fall when released.”

(b) “John ought to be in a lot of pain,”

in the context where John has just suffered an injury.

The focus of this article is on the formal semantic analysis of deontic expressions. We are not so concerned with analysing the ways in which obligations and permissions can be formulated in natural language.

Some uses of deontic expressions may be intended to describe which obligations and permissions are currently in operation. Other uses of deontic expression may actually bring such obligations and permissions into being, as with a proclamation, edict, degree, and other essentially performative utterances and declarations (see Kamp [1973, 1979], Kempson [1977], Lemmon [1962b], for example). We will not consider the performative interpretation of these kinds of speech acts here.

1.2 Formal semantics of natural language

In the paradigm of formal semantics, the aim is to provide a rigorous interpretation of language, and to avoid ad hoc and inconsistent analyses. Usually this is achieved using a systematic translation of natural language into a formal language that has a well defined syntax and behaviour. The translation process and formalisation are often targeted at particular aspects of meaning, and usually do not attempt to deal with the full complexity of meaning in all its richness. This can be seen as a form of abstraction.

In this paradigm, we have to have a clear understanding of our intuitions concerning how a given expression of the language should be interpreted. We need to consider how the behaviour of interest should be captured by the translation process and the formal language, and whether there may be confounding influences from some other aspect of meaning and interpretation.

1.3 Truth conditions and inference

Theories that seek to interpret the propositional content of indicative sentences are concerned primarily with truth. This can involve determining the truth conditions
of indicative “assertoric” sentences, or the legitimate patterns of reasoning from truth to truth.

A logic for deontic expressions needs to concern itself with the validity of claims of the kind given in (4).

(4) (a) The assertion of a deontic statement follows from the assertion of another deontic statement, for example from “You are obliged to mow the lawn and prune the tree” we may infer “You are obliged to mow the lawn”.

(b) Two deontic expressions are incompatible with each other, for example “You must eat all the food” is incompatible with “You must leave some cake for Mary”.

(c) The satisfaction of an obligation is possible. Consider “You must ensure that $1 + 1 = 3$”.

(d) The satisfaction of an obligation implies the satisfaction, or absence of satisfaction, of some other obligation: for example satisfying “You must eat bread and cheese” also satisfies “You must eat bread”.

(e) A particular state of affairs, or action, satisfies an obligation.

(f) A particular state of affairs, or action, is consistent with what is permitted.

For deontic expressions, although we may consider the truth of whether there is an obligation in force, or whether some permission has been granted, it does not automatically follow that the notions of equivalence and entailment for such expressions should necessarily follow the same pattern of inferences that the sentences would have without the deontic modality.

There are, however, many cases where deontic expressions do appear to support patterns of entailment and equivalence akin to those of the corresponding non-deontic propositions. For example, many would agree that from (5a) we should be able to infer (5b), even though the “underlying” propositional content of (5a) “You sit down and eat the cheese” is not being asserted as a straight-forward fact.

(5) (a) “You should sit down and eat the cheese.”

(b) “You should sit down.”
Some deontic systems, such as Standard Deontic Logic (SDL, §2), encapsulate the view that deontic expressions should support the same patterns of entailment as non-deontic expressions. But as we shall see in §4, this is not uncontroversial or unproblematic, as such behaviour appears to lead to a number of dilemmas.

1.4 Obligation to and Obligation that

When it comes to the kind of things that may satisfy an obligation, obvious candidates are actions—where an obligation is an obligation to do something—and outcomes, or states—where an obligation is an obligation that something be the case ([Jackson, 1985]).

In some analyses, the intended interpretation is not made explicit. Furthermore, the boundary between the two characterisations may be somewhat artificial. An action in itself could be characterised by the state of affairs that results from its successful completion (the post-condition of the action).

One approach to this question is simply to ignore it; provided we assume that there is some way of expressing the satisfaction conditions of an obligation, we can go on to consider facets of their analysis without making specific commitments as to their basic nature. Such agnosticism may not always be appropriate: consider (6).

(6) “...surviving being shot is not something that Kennedy ought to have done, though it is something that ought to have been.” ([Jackson, 1985] p179)

1.5 Scope of this article

Here we consider some existing semantic analyses of deontic expressions, and the problems they face. We follow the practice of many working in the field of assuming plausible representations for natural language examples, rather than attempting a rigorous and highly systematic interpretation. While allowing us to focus on the logical and formal details, there is admittedly some danger in this approach: there may be other aspects of interpretation that confound the proposed analysis, or which, if properly analysed, cast light on apparently problematic examples. As we shall see, it is not unusual for formal accounts of deontic expressions to ignore issues such as quantification, predicates and relations, effectively stripping things down to a propositional logic for obligations and permissions.
2 Standard Deontic Logic

Standard Deontic Logic (SDL) represents a classic approach to formalising deontic statements. SDL extends classical propositional logic (see Chapter 5 of Allwood et al., 1977, for example) by adding modal operators (Lemmon and Scott, 1977) for “obligation” and “permission”, together with rules and axioms that govern the behaviour of these new entities (von Wright, 1953). In brief, if \( a \) is a proposition, then \( \text{OB}(a) \) means that \( a \) is obligatory, and \( \text{PE}(a) \) that \( a \) is permitted.

In the literature, the precise syntax for the deontic operators may vary. Another common notation is to use \( O \) for obligation and \( P \) for permission.

2.1 Axioms and Rules for SDL

SDL is conventionally presented using rules and axioms as given in (7), where “\( a \)” and “\( b \)” are propositions, “\( \land \)” represents logical conjunction (and), “\( \lor \)” is disjunction (or), “\( \rightarrow \)” is material implication (if... then...) and “\( \neg \)” is negation (not). The expression “\( \vdash a \)” means that \( a \) is a tautology: the truth of \( a \) follows from the rules and axioms of the logic.\(^2\)

(7) (a) All the axioms and rules of classical logic.
(b) \( \text{OB}(a \rightarrow b) \rightarrow (\text{OB}(a) \rightarrow \text{OB}(b)) \) (OB-K)
(c) \( \text{OB}(a) \rightarrow \neg \text{OB}(\neg a) \) (OB-D)
(d) If \( \vdash a \) then \( \vdash \text{OB}(a) \) (OB-NEC)

Rule 7(b) says that obligation distributes across implication. Rule 7(c) says that if something is obligatory, then its negation cannot also be obligatory. An alternative formulation is \( \neg (\text{OB}(a) \land \text{OB}(\neg a)) \). Finally, 7(d) says that that all tautologies of the logic are obligatory, for example \( \text{OB}(a \lor \neg a) \) for any \( a \).

When taken together, it can be shown that if \( b \) follows from \( a \), then \( \text{OB}(b) \) follows from \( \text{OB}(a) \)—that is, if \( a \) is obligatory, then so is \( b \). This captures the intuition that 5(b) follows from 5(a), for example. This allows the theorems given in (8) to be derived, among other things.

(8) (a) If \( \vdash a \rightarrow b \) then \( \vdash \text{OB}(a) \rightarrow \text{OB}(b) \) (OB-RM)
(b) \( \text{OB}(a \land b) \rightarrow (\text{OB}(a) \land \text{OB}(b)) \) (OB-M)
It is conventional to define permission as the ‘dual’ of obligation, as in (9).

\[(9) \text{PE}(a) \equiv \neg \text{OB}(\neg a)\]

SDL is not uncontentious. It does not impose syntactic or categorial constraints on what kinds of proposition-like entities can appear within the scope of OB and PE (see Castañeda, 1981, for example). To allow the possibility that they might form a distinct class, here \(a, b, \ldots\) are used to denote expressions that can appear as arguments to OB and PE, and \(p, q, \ldots\) will be used to denote regular propositions.

Concerns have also been expressed that SDL is too strong, and that it leads to counterintuitive conclusions, and dilemmas (see §4, and McNamara, 2006a,b, for example). Many authors have expressed concern about (8a) and also (7c), for their role in creating deontic paradoxes and ruling out deontic conflicts, respectively (see Goble, 1990a,b, 1991, 1993; Hansson, 1988, 1990, 2001; Jackson, 1985; Schotch and Jennings, 1980, for example. Some of these issues surrounding (8a) are also discussed by van der Torre, 1997).

### 2.2 A Possible Worlds Model for SDL

In addition to a system of rules and axioms, it is useful to consider whether there is a model that can provide a consistent interpretation of the rules. This can help demonstrate that the proposed rules and axioms are formally coherent. Like many modal logics, SDL can be given a possible worlds interpretation (Kripke, 1959, 1963; von Wright, 1951, 1953). In the standard account, \(a\) is obligatory in the current world, \(\text{OB}(a)\), if and only if \(a\) is true in all accessible ideal worlds, where an ideal world is one in which all obligations have been satisfied. And \(a\) is permitted, \(\text{PE}(a)\), if and only if \(a\) is true in some such worlds.

### 3 Other Approaches

SDL is not the only approach. Here we sketch a small selection of alternative proposals. Additional proposals are discussed in §5.
3.1 The Andersonian-Kangorian reduction

One alternative approach is to say that a proposition is obligatory if some bad thing, a *sanction*, arises whenever that proposition is false, or that this sanction is avoided if the proposition is true. This sanction can be represented by a distinguished proposition $S$.

This approach is proposed by [Prior (1958)](prior) and developed by [Anderson (1958)](anderson). [Kanger (1971)](kanger) gives an equivalent alternative in which the distinguished proposition represents the absence of a sanction. The sanction is fixed, and does not indicate which obligations are unsatisfied.

A variant of this approach, combined with *dynamic deontic logic* ([§3.3](#3.3)) is proposed by [Wyner (2008)](wyner), but where there are propositions that indicate both compliance and non-compliance, and the obligation involved.

3.2 Input–Output Logic

Another alternative to SDL that is founded on different conceptual assumptions is *input–output logic* ([Makinson and van der Torre, 2000, 2001, 2003a,b](makinson_van_der_torre)). Essentially this takes the perspective of an agent that determines what obligations hold on the basis of facts about the state of the world. On this view, a deontic system is an input–output *transducer* from states to obligations. Natural language deontic statements could be interpreted as specifications of this transducer.

3.3 Dynamic Deontic Logic

The final alternative that we will mention here is where obligations, and their satisfaction, are expressed terms of actions within the framework of *dynamic logic* ([Harel, 1984](harel)). We can model actions as things that bring about a state of affairs. Assuming that an action $\alpha$ can be carried out (i.e. its *preconditions* are satisfied), then we can write $[\alpha]p$ to indicate that proposition $p$ is true following the execution of action $\alpha$. Propositions and actions can be combined in various ways.

Using this paradigm, we can follow [Meyer (1988)](meyer), and use OB($\alpha$) to mean that action $\alpha$ is obligatory. It is claimed that this approach can account for problematic examples, such as *Contrary-to-Duty* obligations ([§4.3.4](#4.3.4)), although there may be other problems with this approach (see [Anglberger, 2008](anglberger), for example).
4 Common Issues and Difficulties

There are many problematic examples which present difficulties for formalisations such as SDL. These may be due to: (i) foundational issues concerning whether obligations must be coherent and fulfillable (§4.1); (ii) the use of representations for natural language which have inappropriate consequences (§4.2); and (iii) inappropriate inferential behaviour in the representation language (§4.3). The precise nature of these categories may be subjective and open to dispute. They are not entirely independent, and some examples may have aspects that fall into more than one category. Nevertheless, this categorisation helps to provide some structure to the exposition.

4.1 Foundational Issues

Any account of obligations and permissions has to address the possibility of conflict, either between obligations, and permissions, or between obligations and our understanding of how the world is.

4.1.1 Conflicting obligations

Examples (10) and (11) indicate two cases where there may be conflicting obligations (Lemmon, 1962a).

(10) (a) “You are obliged to have dinner with your friend.”

(b) “You are obliged to rush your choking child to hospital.”

(11) (a) “You are obliged to return the knife.”

(b) “You are obliged to avoid giving a knife to someone who will commit murder.”

Resolving such conflicts may require some way of prioritising or ordering the obligations. It could be argued this is moral rather than a logical question (Bonevac, 1998, p43). Either way, any formal theory of obligations should be able to accommodate conflicts without resulting in inconsistency of the logic itself. This is one motivation for considering alternatives to SDL (§5).
4.1.2 Unfulfillable obligations

We may also question whether all felicitous obligations should be individually fulfillable. Some obligations, such as (12) under a literal interpretation, are clearly unreasonable.

(12) “You are obliged to fly me to the moon.”

Others, such as (13), are not possible.

(13) “Mary must ensure that $2 + 2 = 5$.”

The view that such obligations are infelicitous is characterised by “Kant’s Law”, namely that “ought” implies “can”. This view is not universally accepted (see Martin, 2009, for example). Some argue that Kant’s Law is a conversational implicature rather than a logical rule (Sinnott-Armstrong, 1984).

In general we must account for obligations that conflict with each other, or with the world as we understand it to be, and we should be able to do so without giving rise to a formal inconsistency in the semantic theory itself. This problem of conflict is not confined to deontic expressions.

4.2 Representational Issues

Some seemingly straightforward representations of deontic expressions can have unfortunate consequences. This issue can arise when there is some propositional content—perhaps a relative clause or some propositional condition—that intuitively should lie outside the scope of any obligation.

Some problems might be avoided if the given representations behaved differently, for example if obligations did not distribute to constituent parts (unlike SDL, §2). Even so, there is still an underlying question about how such examples should be represented.

4.2.1 The Good Samaritan

Given one of the obligations in (14), we do not wish to infer that there is an obligation to rob a man in order to then help him, and thus satisfy the obligation to help a robbed man (Prior, 1958).
(14)  (a) “You are obliged to help a man who has been robbed.”
(b) “You are obliged to help a robbed man.”

Such examples are similar to conditional obligations [15]. Indeed, some argue that all forms of the Good Samaritan are essentially disguised conditionals [Castañeda, 1981; Tomberlin, 1981].

(15)  (a) “If a man has been robbed, then you should help him.”
(b) “There is an obligation such that if a man is robbed, you help him.”

Generally, (15a) and (15b) can be formulated in SDL-like languages by an expression of the form (16a) and (16b), respectively, where \( p \) corresponds to “a man has been robbed” and \( a \) is “you help him”.

(16)  (a) \( p \rightarrow \text{OB}(a) \)
(b) \( \text{OB}(p \rightarrow a) \)

It is not clear whether (16b) really expresses what is desired. From this, SDL would allow us to infer (17).

(17) \( \text{OB}(p) \rightarrow \text{OB}(a) \)

This seems odd; (17) states that we are obliged to help not when a robbery has taken place, but in the event that there is an obligation to rob. In the case of (16a), the original conditional obligation (15a) will then be judged “true” in the event that a man has not been robbed.

Various questions can be raised about these representations, such as: the desirability of using material implication to represent conditional obligations (cf. §4.3.4 & §5.2); whether obligations should distribute to constituent parts (§5.4); and whether such inferences should be defeasible (§5.5). The difficulty of analysing complex expressions involving conditionals and other constructs also arises in other non-deontic contexts.

4.2.2 The Knower

Most moral people would argue that from (18) we should not infer (19), given a deontic interpretation of “ought”.

11
It ought to be the case that A knows his wife is committing adultery.

It ought to be the case that A’s wife is committing adultery.

There appears to be a risk of such entailments in some formulations that combine obligation with knowledge (Åqvist, 1967; Jones and Pörn, 1985). This is sometimes called the Paradox of Epistemic Obligation.

4.2.3 The Gentle Murderer

Following Forrester (1984), if we were to utter (20) we probably mean that in the unfortunate event that John murder’s his wife, he ought to do so gently. From this we should not be able to infer (21).

(20) “John ought to murder his wife gently.”

(21) “John ought to murder his wife.”

Other modalities also appear not to distribute into adverbial expressions (Jack- son, 1985). It is unlikely that anyone would claim (23) follows from (22).

(22) “I want to die a painless death.”

(23) “I want to die.”

Jackson (1985) argues that interpretation must be relative to a set of alternatives (see §5.1) as in (24).

(24) Given A (“you murder your wife”) it ought to be the case that B (“you do so gently”).

4.2.4 The Hygienic Cook

Some of the previous conundrums might be avoided if distributive inferences did not apply when faced with contrary obligations (§5.5). But there are examples where such a proposal does not seem entirely appropriate, as in the morally neutral example (25) (Fox, 2010).

(25) “You are obliged to use a clean knife.”
This may give rise to an obligation for the knife to be clean, in contrast to the behaviour of (14). Furthermore, it could be claimed there is no obligation to use a knife (clean or not), only that in the event we use a knife, it ought to be clean, echoing (24).

The different readings available for (25) lend weight to the view that obligations be interpreted with respect to relevant alternatives, as has been proposed for the analysis of the pragmatic notions of topic and focus (Rooth, 1993). This approach has been considered explicitly by Wyner (2008, Section 2.7, pp69–74), in his analysis of the Gentle Murderer (§4.2.3). It appears to correspond to the idea of ‘relativised’ interpretation, as discussed in §5.1.

4.3 Behavioural Issues

Finally in this section, we consider examples that raise questions about the basic behaviour of representations of deontic expressions.

4.3.1 Free choice permission

The issue of free choice interpretations arises with deontic expressions involving disjunction (Kamp, 1973; Ross, 1941), such as (26).

(26) “You may go to the beach or watch television.”

Under the free-choice interpretation (27) the subject can choose which permission to take advantage of.

(27) “You may go to the beach or watch television (or neither), the choice is yours.”

Such free-choice permission may be exclusive (28); if you go to the beach, you may no longer have permission to watch television.

(28) “You may either go to the beach or watch television (or do neither), the choice is yours.”

Free-choice permission appears to indicate a space of possibilities—the “paths” that a subject can take without fear of retribution (Dignum et al., 1996). This interpretation could be captured by considering the consistency (or coherence) of a system of obligations. In particular, (26) would be inconsistent with (29) and perhaps even with (30). This is problematic for SDL, where $\text{PE}(a \lor b)$ follows from $\text{PE}(a)$. 

13
(29) (a) “You are obliged not to go to the beach.”
    (b) “You are obliged not to watch television.”

(30) “You may go to the beach and you may watch television.”

The problem of free-choice disjunction arises in many other contexts, not just those relating to permission (and obligation). Barker (2010) seeks to analyse free choice using the machinery of linear logic, using the insight that linear logic’s resource sensitivity can be used to constrain the extent to which permission has been granted. Others have taken the constraints on entailments to be a matter of implicature rather than logical inference (see Fox, 2007; Franke, 2009; Shulz, 2005, for example), or that free choice should be presented as a choice among epistemic alternatives (see Zimmermann 2000 for example).

4.3.2 Conjunctive commitments

In some cases it may seem that the force of an obligation should distribute across conjunction, as with (5). Given (31) it seems reasonable to conclude both (32a) and (32b).

(31) “You ought to have a shower and go to bed.”

(32) (a) “You ought to have a shower.”
    (b) “You ought to go to bed.”

But consider (33).

(33) “You are obliged to jump off the bridge and land on the train.”

It might be unfortunate if a subject were then to infer (34).

(34) “You are obliged to jump off the bridge.”

Indeed, (33) is presumably consistent with (35).

(35) “It is not permitted for you to jump off the bridge and not land on the train.”

Distributive behaviour is enforced by SDL (8b), but is not supported by other accounts (see Goble 1990a; Jackson 1985; Jones and Pörn 1985; Lewis 1973, for example). This may just be an example of the “and then” sequential interpretation of conjunction, which also arises with indicatives, as in (36).
(36) “John entered the room and turned on the light.”

But a case might still be made that conjunction with obligations can have an “all or nothing” interpretation that is distinct from the sequential interpretation.

Questions about distributive inferences arise with other logical connectives. What will count as an appropriate representation for natural language constructs will depend on which inferences are supported by the chosen logical connectives (§4.2).

4.3.3 Disjunctive Obligations and Ross’s Paradox

Theories that import all valid inferences of classical logic into deontic contexts, like SDL (§2), allow (38) to be inferred from (37).

(37) “You are obliged to post the letter.”

(38) “You are obliged to post the letter or burn the letter.”

One way to satisfy (38) is to satisfy (39).

(39) “You are obliged to burn the letter.”

In general, inferences relating to validity are concerned with deducing what other obligations may also be deemed to be in force, starting from a given system of obligations. This can be contrasted with inferences concerning satisfaction, which seek to deduce what other obligations may be deemed to be satisfied following the satisfaction of one or more obligations.

If judgements concerning validity and satisfaction were conflated, in effect giving a single notion of entailment, then (39) would follow from (37) (Ross, 1945). This counter-intuitive outcome is referred to as Ross’s Paradox.8

The conclusion we can draw from this ‘paradox’ is that the notion of validity (that is, which obligations follow from existing obligations) should not be conflated with the notion of satisfaction (that is, which other putative obligations may be satisfied when satisfying a given obligation).

Even given this distinction, we may wonder whether it is appropriate to be able to infer the obligation (38) from the obligation (37), just as we may question whether the existence of an utterance (or belief) that “p or q is the case” can be inferred
from an utterance (belief) that “p is the case”. One argument against unrestricted
disjunction introduction—exemplified by the move from (37) to (38)—is that there are
free-choice connotations associated with the disjunction which may not be intended.

4.3.4 Contrary to Duty Obligations

Difficulties can arise in analysing obligations that specify how we should make
amends, or compensate, for a failure to satisfy other obligations. A classic example (40) is due to Chisholm [1963].

(40) (a) “It ought to be that a certain man go to the assistance of his neighbours.”
(b) “It ought to be that if he does go, he tell them be is coming.”
(c) “If he does not go then he ought not to tell them be is coming.”
(d) “He does not go.”

From these we should be able to conclude (41).

(41) “He ought not to tell them be is coming.”

It turns out that regardless of whether conditional expressions (40b) and (40c) are
represented in the form $OB(a \rightarrow b)$, or $a \rightarrow OB(b)$ (cf. §4.2.1), apparently faithful
representations of (40a–40d) in SDL are either mutually inconsistent, or one of the
obligations follows from the others. Both of these outcomes run counter to the
intuition that the sentences are mutually consistent and logically independent. Some
proposed solutions are mentioned in §3.3 and §5.1.

Tomberlin (1981) gives a detailed account of the problem of Contrary-to-Duty
obligations, and some possible solutions.

5 Alternative Formalisations

Some of the issues mentioned in §4 have motivated alternative proposals for repre-
senting and reasoning with deontic expressions.

In general, given a straightforward interpretation of deontic statements, SDL ap-
ppears to allow conclusions to be drawn which are counter-intuitive or contradictory.
To avoid this, we may reconsider the nature of the interpretation of natural language
examples (§5.1 and §5.2), prioritise obligations (§5.3) or weaken the logic in some
way (§5.4 and §5.5).
5.1 Relativisation of interpretation

It may be possible to avoid inappropriate patterns of entailment for the Good Samaritan (§4.2.1), the Knower (§4.2.2) and the Gentle Murderer (§4.2.3) by evaluating the meaning of deontic expressions with respect to a context. The obligations to “help” (14), “know” (18), or “murder gently” (20) arise in those contexts in which it is given that there has been (or will be) robbery, adultery, and murder, respectively.

Such ‘relativised’ interpretations have been proposed by Jackson (1985); Kratzer (1981); Prakken and Sergot (1996), for example. Carmo and Jones (2002) disagree with the need to relativise interpretation of deontic expressions in this way, and Zvolenszky (2002) shows there are problems with the relativised account of Kratzer (1981).

5.2 Dyadic modality

The use of dyadic modal operators has been proposed to deal with the conditional forms or interpretations of the Good Samaritan (§4.2.1) (see van Fraassen, 1972, for example), and the Contrary-to-Duty obligations (§4.3.4) (see Prakken and Sergot, 1997, for example). Dyadic modals avoid the use of material implication—as in OB(p → a) and p → OB(a)—and instead borrow the notation of conditional probability, expressing the obligation “to a given that p” by writing OB(a|p) (van Fraassen, 1972; Hansson, 1969; Spohn, 1975; von Wright, 1957).

Dyadic operators can be thought of as relativising obligations (and permissions) to deontic contexts. In these case of the Good Samaritan and Contrary-to-Duty obligations, these may be context in which a man has been robbed, or an obligation is not met.

Appropriate inferential patterns of behaviour can be attributed to dyadic conditionals (Anderson, 1959; Chellas, 1980; van Fraassen, 1972, 1973; von Wright, 1961, 1962), such as (42).

(42) (a) If OB(a|p) and PE(b|p) then OB(a|p ∧ b).

(b) If OB(a|p) and OB(a|q) then OB(a|(p ∨ q)).

The claim formalised by (42a), for example, is intended to capture the idea that bringing about something permitted does not change one’s obligations.
We may model dyadic obligation by saying that $\text{OB}(a|p)$ holds if $a$ is true in the "best" worlds in which $p$ is true. The monadic expression $\text{OB}(a)$ is then equivalent to $\text{OB}(a|\top)$, where $\top$ is a tautology. Other model-theoretic interpretations of dyadic obligation are possible (Hansen, 2008; Hansen et al., 2007). See also Lewis (1974).

5.3 Prioritised obligation

Some dilemmas could be avoided if obligations had different priorities, where higher-level priorities over-rule lower-level priorities (Åqvist, 1967). This could resolve conflicting obligations (§4.1.1), and Contrary-to-Duty obligations (§4.3.4). The issue then becomes how to determine priorities, and indeed whether there should be fixed priorities within the logic. As discussed below in §5.5 there are alternatives for resolving conflicts that may not need to appeal directly to a fixed priority assignment.

In general we may question whether it is the responsibility of a linguistic theory of meaning to account for such behaviour, or whether this falls within the realms of general, non-linguistic reasoning. The problem of conflicts is a general one that also arises with non-deontic utterances.

5.4 Weaker logic

Many deontic dilemmas and conflicts could be resolved by weakening the logic in various ways (see Goble, 1999, 2001, 2004; Routley and Plumwood, 1989, for example). For instance, difficulties with some apparently problematic inferences—like the Good Samaritan (§4.2.1) and conjunction (§4.3.2)—might be resolved if obligations did not distribute across logical connectives such as conjunction (Jackson, 1985; Jones and Pörn, 1985).

If a logic has OB-RM (8a) as a theorem, as is the case with SDL, then obligations will distribute across conjunction; and disjunction introduction within deontic contexts will also follow (§4.3.3). Given that such inferences are sometimes thought to be problematic, some propose weakening the logic so that OB-RM does not follow (Goble, 1990a,b, 1991, 1993; Hansson, 1988, 2001; Jackson, 1985).

Others defend OB-RM on the grounds that it captures the idea of an agent taking moral responsibility for the logical consequences of her commitments (Nute and Yu, 1997; Schotch and Jennings, 1989). But to argue that agents need to understand
the consequences of their obligations does not mean that OB-RM must necessarily be supported [Jackson 1985].

There are proposals for weaker logics that capture salient inferences between obligations, such as the “weakened” OB-RM of Goble (2004), where if $A$ implies $B$, then $OB(A)$ implies $OB(B)$ provided $A$ is permitted.

### 5.5 Weaker inference

An alternative to weakening the rules and axioms of a theory is to adopt a different kind of logic, with a weaker notion of inference. With such an approach, we can still allow obligations to distribute, for example, but take any problematic entailments to be defeasible [Bonevac 1998, Makinson and van der Torre 2003b, Nute 1997]. For the Good Samaritan (§4.2.1) a prior obligation not to rob overrides the default inference to rob, and for the Gentle Murderer (§4.2.3), a prior obligation not to murder overrides the default inference to murder. This may be appropriate if distributive inferences are thought appropriate in “normal” circumstances, and the main issue with the Good Samaritan, and similar examples, is viewed as residing in a conflict between primary obligations and derived obligations. Arguably this is related to proposals to stratify deontic statements into different levels of priority (§5.3). There may be both logical and moral issues to resolve in determining the relative priority of obligations.

The idea of withdrawing conflicting conclusions does not seem to help determine the precise nature of the obligation imposed by the Clean Knife example (§4.2.4), where there are no prior prohibitions on cleaning, or using, a knife.

In the case of deontic conflicts, it is also possible to consider paraconsistency, where reasoning is performed with respect to maximal consistent collections of obligations [da Costa 1988, da Costa and Carnielli 1986, Loparic and Puga 1986].

### 5.6 Logic-free obligations

An alternative approach sketched by Fox (2009) is to allow entailments between the satisfaction conditions of obligations, but not directly between obligations themselves. If an obligation is unsatisfied, than a transgression has occurred. For those obligations that have been satisfied, we may wish to record the subject’s
compliance. Transgressions, and compliance, can be specific to the obligation in question (cf. Wyner 2008), unlike the notion of a single, universal sanction (§3.1).

This allows for partial fulfilment, including partial fulfilment of contradictory and unfulfillable systems of obligations, as well as Contrary-to-Duty obligations (§4.3.4). In such cases, if an agent fails to comply with any compensating obligations then there are simply more unfulfilled obligations (or transgressions). The choice of which obligations to satisfy, and which transgressions to avoid, can then be considered a question of moral judgement, rather than one of logical inference (cf. Bonevac 1998, p43).

A notion of coherence (cf. Makinson and van der Torre 2003b) can be used in place of logical entailment. Instead of $\text{OB}(a)$ following from $\text{OB}(a \land b)$, we can say that a coherent system of obligations will not combine $\text{OB}(a \land b)$ with $\text{OB}(\neg a)$, or indeed with any obligation whose satisfaction is at odds with the satisfaction of $\text{OB}(a \land b)$. If needed, equivalence and subsumption relationships between deontic systems can be formulated in terms of satisfaction conditions and coherence properties.

Coherence can be used to analyse permission. If $a$ is permitted, $\text{PE}(a)$, then it would be incoherent for there to be obligations whose satisfaction is at odds with $a$. For free-choice permission (§4.3.1), if “$a$ or $b$” is permitted, $\text{PE}(a \lor b)$, then it would be incoherent to have obligations that are at odds with $a$, or with $b$. In the case of exclusive free choice, it would be incoherent to combine $\text{PE}(a \lor b)$ with $\text{PE}(a)$ and $\text{PE}(b)$.

By itself, this approach does not resolve how to identify the specific obligations imposed by the Good Samaritan (§4.2.1), the Gentle Murderer (§4.2.3) and the Clean Knife (§4.2.4) examples. They may merit more analysis of the linguistic data, and the use contextualised interpretations (§5.1).

6 Further Reading

McNamara (2006a,b) describes SDL and other approaches, together with discussion of various paradoxes and conundrums and proposals for their resolution. McConnell (2002) discusses some moral dilemmas that any treatment of obligations and permissions should consider. Hansen et al. (2007) presents key philosophical questions.
about deontic logic from the perspective of input/output logic. Other survey papers include Åqvist (2002); Carmo and Jones (2002); Føllesdal and Hilpinen (1971); Hilpinen (1981a); Meyer and Wieringa (1993a).
Notes

1. The terms sentence, statements, utterance and linguistic expression refer to different notions. Not all sentences are statements. Not all utterances or expressions are sentences. And an expression exists independently of any particular physical utterance. In this article we are not especially concerned about such distinctions. One case where the precise terminology is relevant is when the act of making a particular utterance itself imposes an obligation or grants a permission.

2. Here the names of the various rules and axioms (OB-K, OB-D, OB-NEC, etc.) are derived from the conventional names for rules and axioms of these forms in modal logic (K, D, NEC, etc.). The provenance of these names is varied (K for Kripke, D for deontic, NEC for necessitation, etc.). Essentially they are given here as they are part of the vernacular of modern logical theories.

3. Some semanticists might even view the model as not just allowing a demonstration of consistency, but of providing, in some sense, the “real” semantics of the formal account.

4. This kind of approach has been considered for the analysis of imperatives (Lascarides and Asher, 2004; Segerberg, 1990). We will not attempt to consider the relationship between deontic statements and imperatives in this article.

5. Examples of the form (10) and (11) are sometimes referred to, respectively, as Satre’s Dilemma, from Sartre (1957/1946), and Plato’s Dilemma, from Plato’s Republic, Book I “…if a man borrows weapons from a sane friend, and if he goes mad and asks for them back, the friend should not return them, and would not be just if he did. Nor should anyone be willing to tell the whole truth to someone who is in such a state.” (Republic, I, 331c). This example is used to counter the argument that “Justice is speaking the truth and repaying debts.” (Republic, I, 331b–c).

6. Here we use \( p \) to stand for a regular proposition. This is in contrast to \( a \), which may be constrained to be some form of “practive” proposition, in the sense of Castañeda (1981).

7. If \( p \) is false, then material implication allows us to derive \( p \rightarrow \text{OB}(a) \) for any \( a \). This may appear a counter-intuitive interpretation of conditionality. Some propose a
distinct notation for conditional obligation, such as $0B(a|p)$, as sketched in §5.2.

Others have used alternatives to material implication, such as \textit{strong} implication (Prior 1958 for example).

8. Ross’s Paradox was originally described in the context of imperatives.


10. The use of relativised interpretations for deontic expressions appears similar to proposals to use contextually relevant “comparison sets” in the pragmatic interpretation of discourse focus (Rooth 1993).
Works Cited


